1. (10 pts) The position of a body on $0 \leq t \leq 4$ can determined by the function:

$$s(t) = 8t^2 - t + 2$$

(a) (5 pts) Determine the average velocity on the interval $[2, 2 + h]$.

(b) (3 pts) Determine the average velocity on the interval $[2, 2.21]$.

(c) (2 pts) Determine the instantaneous velocity at time $t = 2$.

2. (15 pts) If $f(x) = e^{-x/2} \cos (x)$,

(a) (10 pts) Determine the linearization of the function at $x = 0$

(b) (5 pts) Use the linearization to approximate $f(0.2)$
3. (40 pts) Differentiate the following functions. You do not need to simplify your answer.

(a) \( f(x) = 3(6x + 5)^3(x^2 - 1)^4 \)

(b) \( y = \frac{3xe^{-x}}{5 + 9x^2} \)

(c) \( f(x) = \sin^3(4x) \)

(d) \( y = \ln\left( \sqrt{\frac{x^2 + 7x + 1}{12x - 7}} \right) \)
(e) $f(x) = \tan(4x) + \arctan(4x)$

(f) $y = \left(1 + 3e^{x/6}\right)^{1/x}$

(g) $f(x) = 5x\sqrt{25 - x^2}$

(h) $y = \int_4^{\pi} \cos^4(6u) \sin^4(8u) \, du$
4. (15 pts) Determine the limit (show all work)

(a) \( \lim_{x \to 0} \frac{\ln(e^x + 3 \sin x)}{x + \sin(\pi x)} \)

(b) \( \lim_{x \to \infty} \left( 1 - \frac{1}{2x} \right)^x \)

(c) \( \lim_{x \to 1^-} \frac{\arccos(x)}{\sqrt{1 - x^2}} \)
5. (15 pts) A jogger runs along an elliptic track which has the path

\[80x^2 + 100y^2 = 10500\]

measured in meters. As he reaches the point (10, 5), the \( x \)-coordinate of his path is changing at a rate of 3 m/sec.

(a) (5 pts) At what rate is the \( y \) coordinate changing?

(b) (10 pts) A spectator stands at the point (22, 10). At what rate is the distance from runner to the spectator changing at this time?
6. (20 pts) For the function \( f(x) = \frac{x^3}{x - 2} \) answer the following:

(a) (5 pts) List the critical values of \( f(x) \).

(b) (5 pts) For each listed \( x \) above, state whether there is a local maximum, local minimum, inflection point, or vertical asymptote.

(c) (5 pts) \( \lim_{x \to -\infty} f(x) = \) \( \lim_{x \to \infty} f(x) = \)

(d) (5 pts) Graph the function. Show asymptote(s), intercepts and the exact points (\( x \) and \( y \) coordinates) where the function has any or all local maximum, local minimum values.
7. (15 pts) A cylindrical can is to have volume $62.5\pi$ cm$^3$. If the cost to make the top and bottom lid is $1.00/\text{cm}^2$ and the cost to make the cylindrical side is $2.00/\text{cm}^2$, what are the dimensions of such a cylinder that would minimize cost? 
Note: $v = \pi r^2 h$. Area of each lid is $a_L = \pi r^2$. Area of cylindrical side is $a_S = 2\pi rh$. 
8. (10 pts) Use Newton’s Method once starting with $x_0 = 3$ to approximate the solution to $x = \sqrt{25 - x^2}$.

9. (10 pts) At the point (1, 0), determine the equation of the tangent line to the curve

$$(2x + 3y)^3 - 6x - 12y = 2.$$
10. (30 pts) Evaluate the following integrals.

(a) \[ \int_{0}^{1} \left( \frac{4}{1 + x^2} + \sqrt{8 + x} \right) \, dx \]

(b) \[ \int 3x^4 \ln(x) \, dx \]

(c) \[ \int \frac{x + 1}{x^2 + 2x + 7} \, dx \]
(d) \[ \int \frac{5}{(4x + 5)^3} \, dx \]

(e) \[ \int \sin^2(x) \cos^3(x) \, dx \]

(f) \[ \int 7x \sin(3x) \, dx \]
11. (10 pts) Below is the plot of \( f'(x) = \begin{cases} \sqrt{4-x^2} & 0 \leq x \leq 2 \\ -2 + 2|x-3| & 2 \leq x \leq 5 \end{cases} \)

Use geometry to evaluate the integrals below:

\[
\int_0^2 \sqrt{4-x^2} \, dx = \quad \int_2^5 (-2 + 2|x-3|) \, dx =
\]

12. (10 pts) Sketch the position function in the grid below starting with \( f(0) = 0 \). Show proper concavity of the curve. Give the exact values of the function in the spaces to the right.

\[
f(2) = \quad f(3) = \quad f(4) = \quad f(5) =
\]