1. (10 pts) The position of a body on $0 \leq t \leq 4$ can determined by the function:

$$s(t) = 8t^2 - t + 2$$

(a) (5 pts) Determine the average velocity on the interval $[2, 2+h]$.

(b) (3 pts) Determine the average velocity on the interval $[2, 2.21]$

(c) (2 pts) Deternine the instantaneous velocity at time $t = 2$.

2. (15 pts) If $f(x) = e^{-x/2} \cos(x)$,

(a) (10 pts) Determine the linearization of the function at $x = 0$

(b) (5 pts) Use the linearization to approximate $f(0.2)$

3. (40 pts) Differentiate the following functions. You do not need to simplify your answer.

(a) $f(x) = 3(6x+5)^3(x^2-1)^4$

(b) $y = \dfrac{3xe^{-x}}{5+9x^2}$

(c) $f(x) = \sin^3(4x)$

(d) $y = \ln\left(\sqrt{\dfrac{x^2+7x+1}{12x-7}}\right)$

(e) $f(x) = \tan(4x) + \arctan(4x)$

(f) $y = \left(1 + 3e^{x/6}\right)^{1/x}$

(g) $f(x) = 5x\sqrt{25 - x^2}$

(h) $y = \int_4^{\sqrt{x}} \cos^4(6u) \sin^4(8u)\, du$

4. (15 pts) Determine the limit (show all work)

(a) $\lim_{x \to 0} \dfrac{\ln(e^x + 3\sin x)}{x + \sin(\pi x)}$

(b) $\lim_{x \to \infty} \left(1 - \dfrac{1}{2x}\right)^x$

(c) $\lim_{x \to 1^-} \dfrac{\arccos(x)}{\sqrt{1 - x^2}}$

5. (15 pts) A jogger runs along an elliptic track which has the path

$$80x^2 + 100y^2 = 10500$$

measured in meters. As he reaches the point $(10, 5)$, the x-coordinate of his path is changing at a rate of 3 m/sec.

(a) (5 pts) At what rate is the y coordinate changing?

(b) (10 pts) A spectator stands at the point $(22, 10)$. At what rate is the distance from runner to the spectator changing at this time?

6. (20 pts) For the function $f(x) = \dfrac{x^3}{x-2}$ answer the following:

(a) (5 pts) List the critical values of $f(x)$.

(b) (5 pts) For each listed x above, state whether there is a local maximum, local minimum, inflection point, or vertical asymptote.

(c) (5 pts) $\lim_{x \to -\infty} f(x) =$ ________________ $\lim_{x \to \infty} f(x) =$ ________________

(d) (5pts) Graph the function. Show asymptote(s), intercepts and the exact points (x and y coordinates) where the function has any or all local maximum, local minimum values.

7. (15 pts) A cylindrical can is to have volume 62.5π cm^3. If the cost to make the top and bottom lid is \$1.00/cm^2 and the cost to make the cylindrical side is \$2.00/cm^2, what are the dimensions of such a cylinder that would minimize cost?
Note: $v = \pi r^2 h$. Area of each lid is $aL = \pi r^2$. Area of cylindrical side is $aS = 2\pi rh$.

8. (10 pts) Use Newton's Method once starting with $x_0 = 3$ to approximate the solution to $x = \sqrt{25 - x^2}$

9. (10 pts) At the point $(1, 0)$, determine the equation of the tangent line to the curve

$$(2x + 3y)^3 - 6x - 12y = 2.$$

10. (30 pts)Evaluate the following integrals.

(a) $\int_0^1 \left(\frac{4}{1+x^2} + \sqrt{8+x}\right)\, dx$

(b) $\int 3x^4 \ln(x)\, dx$

(c) $\int \frac{x+1}{x^2+2x+7}\, dx$

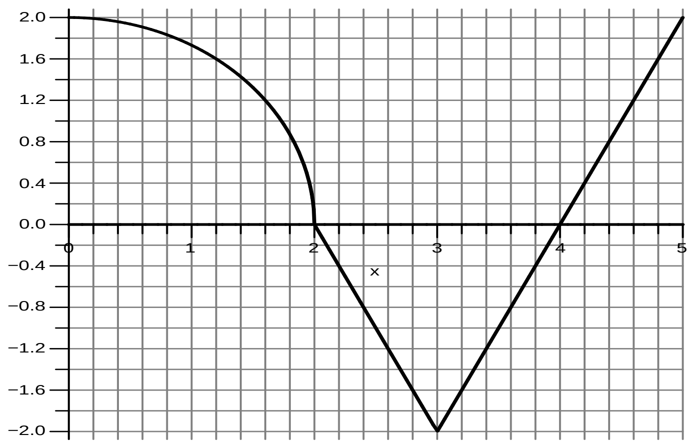
(d) $\displaystyle\int \frac{5}{(4x+5)^3}\,dx$

(e) $\displaystyle\int \sin^2(x)\cos^3(x)\,dx$

(f) $\displaystyle\int 7x\sin(3x)\,dx$

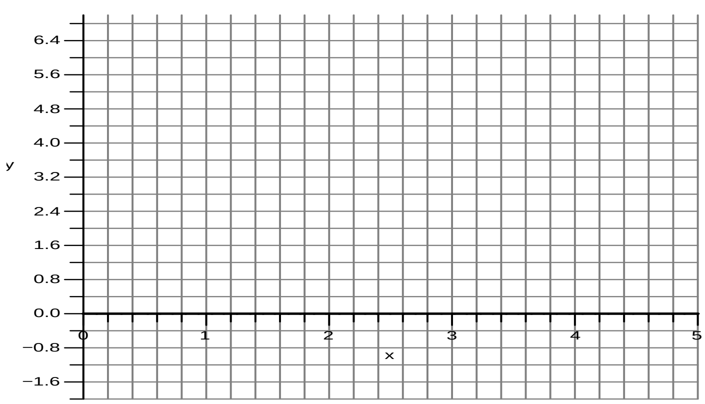
11. (10 pts) Below is the plot of $f'(x) = \begin{cases} \sqrt{4-x^2} & 0 \le x \le 2 \\ -2+2|x-3| & 2 \le x \le 5 \end{cases}$

Use geometry to evaluate the integrals below:



$\int_0^2 \sqrt{4-x^2}\,dx =$ ______ $\qquad \int_2^5 (-2+2|x-3|)\,dx =$ ______

12. (10 pts) Sketch the position function in the grid below starting with $f(0) = 0$. Show proper concavity of the curve. Give the exact values of the function in the spaces to the right.



$f(2) =$ ____________

$f(3) =$ ____________

$f(4) =$ ____________

$f(5) =$ ____________