1. Solve the initial-value problem. Show all the work. Mention a type of the given differential equation.

   (a) $\dfrac{y'}{3} = x^2 y, \qquad y(0) = -8, \quad \text{where } y' = \dfrac{dy}{dx}.$

   (b) $t\dfrac{dx}{dt} = 4x + t^4, \quad x(1) = 5.$

   (c) $y'' - 4y = 2e^{4t}, \qquad y(0) = -3, \quad y'(0) = 11.$

2. Find the general solution to the equation

$$y'' + 4y' + 4y = 12t^2 - 10.$$

3. A 1-kg mass when attached to a spring, stretches the spring to a distance of 4.9 m.

   (a) Calculate the spring constant.

   (b) The system is plased in a viscous medium that supplies a damping constant $\mu = 3$ kg/s. The system is allowed to come to rest. Then the mass is displaced 1 m in the downward direction and given a sharp tap, imparting an instantaneous velocity of 1 m/s in the downward direction. Find the position of the mass as a function of time.

4. Use Laplace transform to solve the IVP

$$y'' - y = e^t \cos t, \qquad y(0) = y'(0) = 0.$$

5. Find the Laplace transform of the function

$$g(t) = \begin{cases} 3t & \text{for } 0 \le t < 2 \\ 4 & \text{for } t \ge 2 \end{cases}$$

6. Find the unit impulse response to the initial-value problem

$$y'' - 2y' + 5y = \delta(t), \qquad y(0) = y'(0) = 0$$

7. For the initial-value problem $y' = y + 4t$, $y(0) = 1$ calculate the first two iterations of Euler's method with step size $h = 0.1$.

8. Write the second-order equation as a system of two first-order equations

$$y'' - e^{-2t} + 3t^2 y = \cos t y'.$$

9. Find the general solution to the system. Write the answer in a vector form.

$$y_1' = -3y_1 - 6y_2$$
$$y_2' = -y_2$$

10. By using the variation of parameters technique and the fundamental matrix find a particular solution to the system

$$y_1' = -3y_1 - 6y_2 + 2e^{-5t}$$
$$y_2' = -y_2$$

You may use results from the previous problem.

11. For the nonlinear system

$$x' = x(4y - 5)$$
$$y' = y(3 - x)$$

find all equilibrium points, classify their types and determine stability (stable, unstable or asymptotically stable).

12. Expand the given function in a Fourier cosine series valid on the interval $0 \le x \le \pi$. Calculate a_0 separately.

$$f(x) = x.$$

13. Find the temperature $u(t, x)$ in a rod modeled by the initial/boundary value problem

$$u_t = 0.03\, u_{xx}, \quad \text{for } t > 0, \;\; 0 < x < \pi,$$
$$u_x(0, t) = u_x(\pi, t) = 0, \quad \text{for } t > 0,$$
$$u(x, 0) = x, \quad \text{for } 0 \le x \le \pi.$$

You may use results obtained in the previous problem.