

MATH 0230 - Analytic Geometry and Calculus II
SAMPLE FINAL EXAM 2

Time: 1 hour 50 min

1. Evaluate the integral if it converges. If it diverges, show divergence.

(a)
$$I = \int_{-\infty}^0 xe^{-3x^2} dx$$

(b)
$$I = \int_3^7 \frac{5}{(3-x)^3} dx$$

2. At what points on the curve $x = 2t^3$, $y = 3t^2 + 9t - 5$ does the tangent line have slope $\frac{1}{2}$?

3. Find the area of the region that lies inside the curve $r = 3 \sin \theta$ and outside the curve $r = 1 + \sin \theta$.

4. Find the volume generated by rotating the region bounded by the curves $x = y^2$ and $x - 3y + 2 = 0$ about the y -axis.

5. Find the volume of the parallelepiped with adjacent edges PQ , PR , and PS if $P(3, 0, 1)$, $Q(-1, 2, 5)$, $R(5, 1, -1)$, and $S(0, 4, 2)$.

6. Find parametric equations for the line through $(2, 4, 6)$ that is perpendicular to the plane $x - y + 3z = 7$. In what points does the line intersect the coordinate planes?

7. A water tank is in the shape of an inverted cone (with its sharp end down). The height of the cone is 10 meters and the diameter of the base is 8 meters. The tank is full of water. Find the work required to pump all of the water out over the side. You may use $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$ for the density of water and $g = 9.8 \frac{\text{m}}{\text{s}^2}$ for the acceleration due to gravity.

8. Determine whether the series is convergent or divergent. Clearly show reason by a valid test.

$$(a) \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{\sqrt{n^2 + 3}}$$

$$(b) \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n (n + 3)$$

9. For the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{2n}{n^2 + 1}$$

determine whether it is absolutely convergent, conditionally convergent or divergent. Clearly show reason by a valid test.

10. Determine the sum of the given series.

$$(a) \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n}$$

$$(b) \sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$$

11. Determine the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n \sqrt{n}}$$

12. For the functions below, determine their Maclaurin series. You may apply any general series you know. You should use sigma \sum notation.

(a) $\frac{3}{(1-3x)^2}$

(b) $x e^{-3x}$

13. For the functions below, determine their Taylor series about given values of a . You may apply any general series you know. You should use sigma \sum notation.

(a) $f(x) = x^4 - 5x^2 + 8x - 4, \quad a = 1$

(b) $\cos 2x, \quad a = \frac{\pi}{4}$

14. Solve the initial-value problem. Show all the work. Mention a type of the given differential equation.

$$(a) \quad y' = \frac{\sin x}{2y}, \quad y(\pi) = 2.$$

$$(b) \quad y' + \frac{3}{x}y = \frac{2}{x^2}, \quad y(1) = 5.$$

$$(c) \quad y'' - 2y' + 10y = 0, \quad y(0) = 0, \quad y'(0) = 6.$$

15. Find the general solution to the second-order nonhomogeneous differential equation

$$y'' + 4y = 8 \cos 2x$$

Use the method of undetermined coefficients.