MATH 0230 - Analytic Geometry and Calculus II
SAMPLE FINAL EXAM 2

Time: 1 hour 50 min

1. (10 points) Evaluate the integral $I = \int_0^{\pi/3} \sin^3 x \cos^2 x \, dx$.

2. Evaluate the integral if it converges. If it diverges, show divergence.

   (a) (10 points) $I = \int_3^7 \frac{1}{\sqrt{7-x}} \, dx$

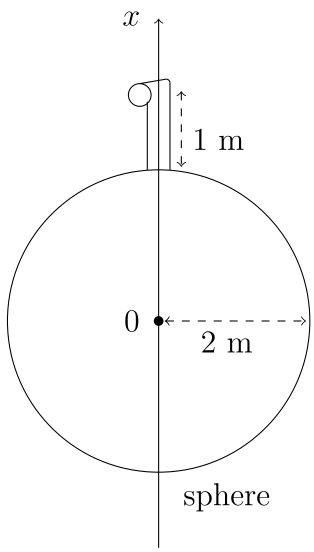
   (b) (10 points) $I = \int_1^\infty \frac{x}{4+x^2} \, dx$

3. (10 points) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = x^2$ and $y = 4x$ about the y-axis. No credit will be given if a different method is used.

4. (10 points) A water tank has the shape of a sphere of radius 2 m with a spout at the top of length of 1 m, as in the following picture below. The spherical part of the tank is full of water and the spout is empty.

Set up **but do not evaluate** an integral to find the work required to empty the tank by pumping all of the water out of the spout. You may use $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$ for the density of water and $g = 9.8 \frac{\text{m}}{\text{s}^2}$ for the acceleration due to gravity.

Use the direction of the x-axis and the origin provided in the picture.



5. For the given points $P(1, -3, 1)$, $Q(2, -2, -1)$, and $R(-2, -3, 4)$.

(a) (5 points) find the angle θ between vectors $\bar{\mathsf{a}} = \bar{\mathsf{PQ}}$ and $\bar{\mathsf{b}} = \bar{\mathsf{PR}}$. Leave the answer in an exact form using $\cos^{-1}$.

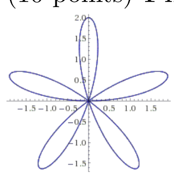
(b) (5 points) find the area of the triangle PQR.

(c) (5 points) find a linear equation of the plane that passes through the points P, Q, and R.

(You may use results found in previous parts).

6. (10 points) Find the equation of the line tangent to the curve $x = \sin t,\ \ y = (t+1)^2$ at the point where $t = 0$.

7. (a) (10 points) Find the area enclosed by one leaf of the graph of $r = 2 \sin 5\theta$.



(b) (5 points) Set up **but do not evaluate** an integral to find the length of the boundary of one leaf of the graph.

8. For series in parts (a) and (b) below, determine whether the series is convergent or divergent. Clearly show reason by a valid test. If it is convergent, find its sum.

(a) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5^n}$

(b) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(c) (5 points) For the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$ determine whether it is convergent or divergent.

If the series is convergent, then DO NOT find its sum.

9. Determine the sum of the given series.

(a) (10 points) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2 + 3n + 2}$

(b) (5 points) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{5^n\, n!}$

10. (10 points) Determine the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n\, x^n}{n\, 4^n}$$

11. For the functions below, determine their Taylor series about $x = 0$. You may apply any general series you know. You should use sigma $\sum$ notation.

(a) (5 points) $\dfrac{1}{5+x}$

(b) (5 points) $\ln(5+x)$

(c) (5 points) $-x^2\cos(2x)$

12. (10 points) Find the Taylor series for the function $f(x) = \sin x$ about $a = \pi$.

You should use sigma $\sum$ notation.

13. Solve the initial-value problem. Show all the work. Mention a type of the given differential equation.

(a) (10 points) $\frac{y'}{3} = x^2 y, \qquad y(0) = 5.$

(b) (10 points) $xy' - 2y = x^3, \qquad y(1) = 4.$

(c) (10 points) $y'' + 6y' + 9y = 0, \qquad y(0) = 2, \quad y'(0) = -5.$

14. (15 points) Find the general solution to the second-order nonhomogeneous differential equation by using the method of undetermined coefficients.

$$y'' + 2y' + 5y = -16x\, e^{x}$$