Talk 2: New families of nonreflexive Banach spaces with the fixed point property

In 2008 P. K. Lin proved the existence of an equivalent norm $|||\cdot|||$ in the sequence space ℓ_1 such that $(\ell_1, |||\cdot|||)$ verifies the FPP. P.K. Lin's paper opened new vias of research since $(\ell_1, |||\cdot|||)$ was the first known non-reflexive Banach space with the FPP. Since then, P. K. Lin and some other authors have proved the existence of new equivalent norms in ℓ_1 with the FPP. In fact, P.K. Lin's result strongly relies on the following property for the usual norm in ℓ_1:

$$\limsup \|x_n + x\|_1 = \limsup_n \|x_n\|_1 + \|x\|_1$$

for every w^*-null sequence $(x_n) \subset \ell_1$ and $x \in \ell_1$.

In this paper we define the concept of sequentially separating norm in a Banach space with a Schauder basis which is more general than the previous equality. We will prove that every Banach space with a boundedly complete Schauder basis which admits a sequentially separating norm can be renormed to have the FPP. In fact, we will prove that in this case, the set of equivalent norms with the FPP contains n-dimensional affine manifolds for every $n \in \mathbb{N}$.

We will apply our techniques to different classes of Banach spaces and we develop a method to construct nonreflexive Banach spaces which are FPP-renormable. We will illustrate the scope of our techniques by showing several examples.

Finally, we will study some geometric properties for Banach spaces which can be renormed with a sequentially separating norm. In particular we deduce that these Banach spaces verify the Schur property and they are hereditarily ℓ_1 (properties which do not depend on the equivalent norm which is considered).