For X a Banach space we denote by $\mathcal{P}(X)$ the set of all equivalent norms in X.

Talk 1: Stability of the Fixed Point Property

Fixed Point Property (FPP) is not an isomorphic property, that is, it is not preserved it we change the norm by an equivalent one. However for certain classes of Banach spaces, the FPP can be inherited by equivalent norms which are "close" enough to the previous one. This happens to the most classical reflexive Banach spaces. What is more, it can be proved that for every separable reflexive Banach space there exists an equivalent norm which produces stability of the FPP.

Just the opposite happens for classical nonreflexive Banach spaces. It is well-known that the sequence Banach spaces ℓ_1 and c_0 fail to have the FPP when they are endowed with their usual norms. While it is an open problem whether c_0 can be renormed to have the FPP, it is known that there exist some equivalent norms that make ℓ_1 satisfy the FPP. However, given any equivalent norm in either ℓ_1 or c_0, we can always find another one, as close to the first as we like, and without the FPP, that is: no norm in ℓ_1 and c_0 can produce stability of the FPP. In other words, this means that the set of equivalent norms in ℓ_1 and c_0 which fail the FPP is dense in $\mathcal{P}(\ell_1)$ and $\mathcal{P}(c_0)$ respectively. We will focus on equivalent norms in c_0 which fails to have both an asymptotically copy of c_0 and the FPP. We will prove that this set is also dense in $\mathcal{P}(c_0)$ and we will state some open problems related to the similar case for ℓ_1.

- Can c_0 be renormed to have the FPP?
- Does there exist any equivalent norm in ℓ_1 which fails both the FPP and to have an asymptotically isometric copy of ℓ_1?
- It is known that ℓ_1 can be renormed to have the FPP. However, is the set of equivalent norms in ℓ_1 with the FPP dense on $\mathcal{P}(\ell_1)$?