

IRREGULAR CONVEX SETS WITH FIXED POINT PROPERTY

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ABSTRACT. It is well known that the fixed point property for a c.b.c set \mathcal{C} depends strongly on the nice geometrical properties of the Banach space X or on the set \mathcal{C} itself. For example, among c.b.c sets having f.p.p are all compact sets, all sets in uniformly convex spaces, all weakly compact sets having normal structure etc.

We will construct some examples of very irregular sets in l_1 , not satisfying the above conditions, but still having f.p.p. The method is based on the notion of asymptotic center. Together with the nice properties of weak-star convergence in l_1 we will see that the intersection of two sets having f.p.p may not have f.p.p or that there exists a sequence of sets $\{C_n\}$ such that $C_1 \supset C_2 \supset \dots \supset C_n \dots$ and also C_n has f.p.p for $n = 1, 3, 5, \dots$ and fails f.p.p for $n = 2, 4, 6, \dots$