

REGULARITY OF ALMOST MINIMIZERS FOR A DEGENERATE ONE-PHASE BERNOULLI-TYPE FUNCTIONAL

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In this talk, we deal with almost minimizers for the energy functional

$$(1) \quad J_p(u, \Omega) := \int_{\Omega} \left(|\nabla u(x)|^p + \chi_{\{u>0\}}(x) \right) dx, \quad p > 1,$$

where Ω is a bounded domain in \mathbb{R}^n and $u \geq 0$. The functional J_p is a generalization to each $p > 1$ of the classical one-phase (Bernoulli) energy functional, corresponding to $p = 2$ in (1).

Almost minimizers of J_2 were investigated recently in [2, 1]. However, in [4] D. De Silva and O. Savin provided a different approach with respect to [2, 1], based on nonvariational techniques, to deal with almost minimizers of J_2 and their free boundaries. Precisely, inspired by [5], they showed that almost minimizers of J_2 are “viscosity solutions” in a more general sense. This property roughly means that almost minimizers satisfy comparison in a neighborhood of a touching point whose size depends on the properties of the test functions. Once this property was established, the regularity of the free boundary for almost minimizers followed via the techniques developed by De Silva in [3].

In this talk, we present an optimal Lipschitz continuity result for almost minimizers of J_p , $p > \max \left\{ \frac{2n}{n+2}, 1 \right\}$. Our approach is inspired by the method introduced in [4]. The talk is based on a joint work with S. Dipierro, F. Ferrari, and E. Valdinoci, see [6].

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