A NEW PROOF OF THE STRUCTURE OF GEODESICS IN HIGHER DIMENSIONAL HEISENBERG GROUPS

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Given a geodesic $\gamma : [0, L] \to \mathbb{H}^1$, one can use a simple argument involving the isoperimetric inequality to show that the projection of the curve to the xy-plane must be an arc of a circle. The argument stems from the definition of the Sub-Riemannian metric in the Heisenberg group and from the fact that the symplectic area enclosed by the projected curve must be equal to the curve's net change in the *t*-coordinate. The equations for geodesics into \mathbb{H}^n are well known, and they show that the projection of a geodesic to each $x_i y_i$ -plane is an arc of a circle. However, no proof of this fact exists using a simpler argument involving the Isoperimetric inequality. In this presentation, I will show such a proof.