## WEAK COMPACTNESS IS NOT EQUIVALENT TO THE FIXED POINT PROPERTY IN c

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ABSTRACT. We discuss recent joint work of Gallagher, Lennard and Popescu, where we show that there exists a non-weakly compact, closed, bounded, convex subset W of the Banach space of convergent sequences  $(c, \|\cdot\|_{\infty})$ , such that every nonexpansive mapping  $T: W \longrightarrow W$  has a fixed point. This answers a question left open in the 2003 and 2004 papers of Dowling, Lennard and Turett.

This is also the first example of a non-weakly compact, closed, bounded, convex subset W of a Banach space X isomorphic to  $c_0$ , for which W has the fixed point property for nonexpansive mappings.

We also prove that the sets W may be perturbed to a large family of non-weakly compact, closed, bounded, convex subsets  $W_q$  of  $(c, \|\cdot\|_{\infty})$  with the fixed point property for nonexpansive mappings; and we discuss similarities and differences with work of Goebel and Kuczumow concerning analogous subsets of  $\ell^1$ .

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