ANALYSIS SEMINAR

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Title: The Liouville theorem for averaging operators on grids

Abstract: The interest in harmonic functions on graphs goes back to the nineteenth century, closely related to electrical networks and random walks. Discrete Potential Theory is nowadays an active field, with connections and applications to different areas of pure and applied mathematics.

Motivated by the continuous p-laplacian, some discrete counterparts have been considered, like the *discrete p-laplacian* and, more recently, the so called *game p-laplacian.* In such cases, the corresponding solutions satisfy a local mean value property of the type $u(x) = F(u(x_1), ..., u(x_d))$ where $x_1, ..., x_d$ are the neighbours of x. F is called an *averaging operator* on the graph.

Harnack and Liouville properties are also central topics in Discrete Potential Theory. In the case of the discrete p-laplacian, the Harnack(and therefore Liouville) property was established by Holopainen-Soardi (1997) under certain geometrical assumptions on the graph. Their method is, however, quite indirect because it follows the continuous road, with a Cacciopoli-type inequality, the De-Giorgi-Moser iteration method and a discrete version of the John-Nirenberg lemma as the key ingredients. Holopainen and Soardi suggested the convenience of a more direct argument, only based on the local formulation.

In the talk we will report an elementary compactness proof of Liouville theorem for averaging operators on the grid $\mathbb{Z}^d$, including the cases of the discrete and game p-laplacians. (Joint work with T. Adamowicz).