Deformation of Alexander Maps

We discuss a dimension-free deformation theory for Alexander maps and its applications.

In 1920, J. W. Alexander proved that every closed orientable PL (piecewise linear) n-manifold can be triangulated so that any two neighboring n-simplices are mapped to the upper and the lower hemispheres of $\mathbb{S}^n$, respectively. Such maps are called Alexander maps. Rickman introduced a powerful 2-dimensional deformation method for Alexander maps, in his celebrated proof (1985) of the sharpness of the Picard theorem in $\mathbb{R}^3$.

The higher dimensional topological deformation leads to a Hopf degree theorem for Alexander maps, and a Berstein–Edmonds-type existence theorem for branched covers between manifolds with boundary. The geometrical deformation may be used to extend an example of Heinonen -Rickman on wildly branching quasiregular maps, from dimension 3 to 4. (This is joint work with Pekka Pankka.)