"CHARACTERIZING GEOMETRIC AND TOPOLOGICAL PROPERTIES OF SETS AND SPACES VIA FIXED POINT PROPERTIES."

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Abstract. A major theme in Fixed Point Theory is characterizing geometric and topological properties of sets or spaces in terms of fixed point properties. E.g., from Brouwer (1912), Schauder (1930) and Klee (1955), we know that closed and convex subsets K of a Banach space X are norm compact if and only every continuous map on K has a fixed point. Also, Jaggi (1983) and Kassay (1986) proved that a reflexive Banach space X has normal structure if and only if every Jaggi nonexpansive map on a closed bounded convex subset of X has a fixed point. And Maurey (1980) and Dowling-L (1997) showed that for a subspace X of $(L^1[0,1], \|\cdot\|_1)$, X is reflexive if and only if every nonexpansive map on a closed bounded convex subset of X has a fixed point. Moreover, by Maurey (1980) and Dowling-L-Turett (2004), a closed bounded convex subset K of $(c_0, \|\cdot\|_\infty)$ is weakly compact if and only if every nonexpansive map on K has a fixed point. And L-Nezir (2014), using a theorem of Domínguez Benavides (2009), proved that a Banach lattice $(X, \|\cdot\|)$ is reflexive if and only if X has an equivalent norm $\|\cdot\|^\sim$ such that every $\|\cdot\|^\sim$-cascading nonexpansive map has fixed point. We will discuss the above, and recent developments related to this circle of ideas.