# A SIMPLE PROOF OF EULER'S FORMULA FOR $\zeta(2k)$ AND SOME RATIONAL SERIES REPRESENTATIONS INVOLVING EVEN ARGUMENTS OF THE RIEMANN ZETA FUNCTION.

The Riemann zeta function is defined by

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}, \quad \mathrm{Re}\, s>1.$$

Clearly, the series is absolute convergent. It is well-known that $\zeta$ is analytic and it has an analytic continuation at $s=1$.

Moreover, the Riemann zeta function has the following integral representation,

$$\zeta(s)=\frac{1}{\Gamma(s)}\cdot\int_0^{\infty}\frac{u^{s-1}}{e^u-1}\,du, \quad \mathrm{Re}\, s>1,$$

where $\Gamma(s)=\int_0^{\infty}e^{-t}t^{s-1}dt$, $\mathrm{Re}\, s>0$ is the Euler Gamma function.

In 1734, Leonhard Euler proved (the Basel problem) that

$$\zeta(2)=\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}.$$

Later on (more exactly in 1740), Euler generalized the above result and computed the values of the zeta function at even positive integers. In fact, he proved the following

THEOREM 1.

$$\text{(E):}\quad \boxed{\zeta(2k)=\sum_{n=1}^{\infty}\frac{1}{n^{2k}}=\frac{(-1)^{k+1}\cdot 2^{2k-1}\cdot B_{2k}}{(2k)!}\cdot\pi^{2k}.}$$