VARIABLE LEBESGUE SPACES: AN OVERVIEW, GEOMETRY AND FIXED POINT THEORY

MARIA A. JAPÓN

Given (Ω, Σ, μ) a measure space, the classical Lebesgue space $L^p(\Omega)$ $(1 \leq p < +\infty)$ is defined to be the collection of all measurable functions f such that $\int_\Omega |f(x)|^p d\mu < +\infty$. Assume that p is changed by a variable exponent function $p(x)$, that is, $p : \Omega \to [1, +\infty)$ is now a measurable function that takes on the role of the constant exponent p. The study of the functions f such that $\int_\Omega |f(x)|^{p(x)} d\mu < +\infty$ leads to the definition of Variable Lebesgue Spaces, usually denoted by $L^{p(\cdot)}(\Omega)$.

Although Variable Lebesgue Spaces can be included within the larger family of Musielaz-Orlicz spaces, during the past three decades many researches have felt very much attracted to this variable extension of Lebesgue spaces by their own right. In particular, this field has experimented a burgeoning development in recent times, due in part to the discovery of many applications of $L^{p(\cdot)}(\Omega)$ to PDEs connected with different problems of physical nature.

This seminar will consist of two talks: In the first one we will introduce a general overview of Lebesgue Variable Spaces. It is of great importance to emphasise that some of the standard properties of Lebesgue spaces are not preserved under this variable approach. We will try to bring to light some of the main differences and similitudes of Variable Lebesgue Spaces with respect to the classical L^p spaces. Furthermore, we will focus on those geometric properties related to the theory of the fixed point property for nonexpansive operators.

In the second session we will expose several results concerning the existence of fixed points for nonexpansive mappings defined on closed convex bounded subsets of Variable Lebesgue Spaces $L^{p(\cdot)}(\Omega)$. The behaviour of the exponent function $p(\cdot)$ will become essential in determining the fulfilment of the (weakly) fixed point property. We will check that our results resemble those already known for the standard Lebesgue L^p spaces. Nevertheless, we will find out some pleasing and unexpected surprises.

(Maria A. Japón) Mathematical Analysis Department, University of Sevilla, Spain

E-mail address: japon@us.es