FUNCTIONAL ANALYSIS MINI-CONFERENCE AND SEMINAR, PITT MATH DEP'T

An event associated with the visit of Prof. Maria Japón Pineda, University of Sevilla, Spain.

Saturday 15/February/20, 427 Thackeray, 8:45 am - 5:00 pm.

MARIA'S VISIT IS SUPPORTED BY THE MATHEMATICS RESEARCH CENTER (MRC) IN THE DEPARTMENT OF MATHEMATICS AT THE UNIVERSITY OF PITTSBURGH

***** Coffee and breakfast: 8:45 am - 9:35 am, in 428 Thackeray.**

(1) 9:35 am-10:25 am: **Mr. Adam Stawski, Pitt:**
"Hyperconvex Metric Spaces and the Fixed Point Property."

(2) 10:40 am-11:30 am: **Prof. Maria Japón Pineda, U. Sevilla:**
"Some classical non-reflexive Banach spaces with the Fixed Point Property (and without renormings)."

***** Coffee Break : 11:40 am-12:05 pm, 428 Thackeray.**

(3) 12:05 pm-12:55 pm: **Prof. Ka-Sing Lau, Pitt and CUHK:**
"Dirichlet forms and critical exponents on fractals."

***** Lunch : 1:05 pm-2:40 pm.**

(4) 2:40 pm-3:30 pm: **Prof. Armin Schikorra, Pitt:**
"Knot energies and harmonic maps into spheres."

(5) 3:45 pm-4:35 pm: **Ms. Pamela Delgado, Pitt:**
"Cesàro averaging and new invariant Banach limits on ℓ^∞ and $L^\infty(0,\infty)$."

Abstracts:

Hyperconvex Metric Spaces and the Fixed Point Property

Functional Analysis Mini-Conference and Seminar
University of Pittsburgh 2020
Adam Stawski, University of Pittsburgh

Abstract: In 1979, R. Sine and PM. Soardi independently proved that bounded, hyperconvex metric spaces have the fixed point property for nonexpansive maps. In this survey talk, we will briefly discuss a few results in the theory of hyperconvex metric spaces related to fixed point theory. We will also discuss the context of Soardi's paper, which in particular, gives a proof that the ball of $\ell^\infty(\mathbb{N})$ has the fixed point property for nonexpansive maps.

SOME CLASSICAL NON-REFLEXIVE BANACH SPACES WITH THE FIXED POINT PROPERTY (AND WITHOUT RENORMINGS)

MARIA A. JAPÓN

A Banach space X is said to have the Fixed Point Property (FPP) if every nonexpansive mapping $T : C \to C$, where C is a closed convex bounded subset of X, has a fixed point. Recall that a mapping $T : C \to C$ is nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for every $x, y \in C$. This definition strongly depends on the norm to the extent that, when the norm is replaced by any other equivalent norm, the family of nonexpansive mappings is highly likely to be altered (the FPP is an isometric property but fails to be isomorphic).

In the 60's, it was conjectured that FPP and reflexivity could be equivalent properties for Banach spaces. This conjecture was grounded on the fact that "almost" every classical nonreflexive Banach space was known to fail the FPP. Furthermore, using some additional assets, "every" classical reflexive Banach space was shown to have the FPP.

So far, it is unknown whether every reflexive Banach space enjoys the FPP (my feeling is that there must exist some weird reflexive Banach space with a very bizarre geometry lacking the FPP). Nevertheless, in 2008, P.K. Lin [3] proved that the sequence space ℓ_1 could be renormed to have the FPP, bringing to light that the previous conjecture was false (at least in one direction) and opening new bridges between Renorming Theory and Fixed Point Theory.

Since P.K. Lin's result, some other nonreflexive Banach spaces have been renormed to have the FPP [2] and some amusing norms have been constructed (failing to have asymptotically isometric copies of ℓ_1) enlarging the family of nonreflexive Banach spaces with the FPP [1].

In this talk we will prove that a classical nonreflexive sequence space endowed with its standard norm does satisfy the FPP and we will pose some open questions that can be raised in this regard.

References

[1] E. Castillo-Santos, P.N. Dowling, H. Fetter, M. Japón, C.J. Lennard, B. Sims, B. Turett. *Near-infinity concentrated norms and the fixed point property for nonexpansive maps on closed, bounded, convex sets.* J. Funct. Anal. 275 (2018), no. 3, 559-576.

[2] C.A. Hernández-Linares, M. Japón. *A renorming in some Banach spaces with applications to fixed point theory.* J. Funct. Anal. 258 (2010), no. 10, 3452-3468.

[3] P. K. Lin, *There is an equivalent norm on ℓ_1 that has the fixed point property.* Nonlinear Anal., 68 (8) (2008), 2303-2308.

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Dirichlet Forms and critical exponents on Fractals

by

Ka-Sing Lau

University of Pittsburgh, and The Chinese University of Hong Kong

Abstract: For a domain Ω in $\mathbb{R}^n$ with smooth boundary, it is well-known that the Sobolev spaces $W^{1,2}(\Omega)$ and $W^{s,2}(\Omega), 0 < s < 1$ are function spaces that are associated with the Laplacian Δ and the fractional Laplacian $(-\Delta)^s$ respectively. In the analysis on fractals, these concepts have been extended to local and nonlocal regular Dirichlet forms on fractal sets K, and the associated function spaces on K are Besov spaces $B^{\sigma^*}_{2,\infty}$ and $B^{\sigma}_{2,2},\ 0 < \sigma \le \sigma^*$. We call σ^* the *critical exponent* of the family of Besov spaces. In this talk, we will discuss some of the recent developments; in particular on the Dirichlet forms on $B^{\sigma^*}_{2,\infty}$. We will consider the convergence of the Besov norms $||\cdot||_{B^{\sigma}_{2,2}}$ to $||\cdot||_{B^{\sigma^*}_{2,\infty}}$, analogous to a classical theorem of Bourgain-Brezis-Mironescu on Sobolev spaces. This is a joint work of Dr. Qingsong Gu.

Armin Schikorra

Title: Knot energies and harmonic maps into spheres.

Abstract: I will report on advances in the regularity theory for minimizers and critical points of a class of knot energies defined by Jun O'Hara. When parametrized by arclength the tangent field of these knots are critical points of a $W^{1/p,p}$-type energy, and we employ arguments from the regularity theory of $W^{1/p,p}$-harmonic maps into the sphere. Joint work with S. Blatt, Ph. Reiter.

Cesàro averaging and new invariant Banach limits on ℓ^∞ and $L^\infty(0,\infty)$

Pamela Delgado[1], Chris Lennard[1], Jeromy Sivek[2]
[1]University of Pittsburgh
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Abstract

On the sequence space ℓ^∞, we construct Banach limits that are invariant under the Cesàro Averaging operator. On the function space $L^\infty(0,\infty)$, we start by defining a new operator J^α, for each $\alpha > 0$. This new operator extends the definition of J^n, with $n \in \mathbb{N}$, which is the operator obtained by composing the Cesàro Averaging operator with itself n times. We show that the family of operators $\{J^\alpha\}_{\alpha>0}$ has the semigroup property, and we also construct Banach limits that are invariant under these operators.