Losing compactness in Brouwer's Fixed Point Theorem

Łukasz Piasecki
Instytut Matematyki, Uniwersytet Marii Curie-Skłodowskiej
Pl. Marii Curie-Skłodowskiej 1, 20-031 Lublin, Poland
`piasecki@hektor.umcs.lublin.pl`

Since 1983, when the work of Benyamini and Sternfeld appeared, it is well known that in every infinite dimensional Banach space X the unit sphere S_X is a lipschitzian retract of the unit ball B_X. Moreover, in 1985 Lin and Sternfeld obtained the strongest result in this matter. Indeed, they proved that for any bounded, closed, convex and noncompact set C in X, and for any $k > 1$, there exists a mapping $T : C \to C$ satisfying the Lipschitz condition with the constant k and such that $\inf \{\|x - Tx\| : x \in C\} > 0$. The above mentioned results have their roots in efforts to extend the classical Brouwer's Fixed Point Theorem to noncompact settings. In spite of the fact that they provide qualitative answers to the theory, there are still some quantitative aspects, which were initiated by Goebel in 1973. The two basic are the minimal displacement problem and the optimal retraction problem. The aim of my talk is to give the updated presentation of the subject and possibly attract newcomers to the field.