Presentation : Gromov Compactness Theorem

Keywords: almost complex structure, hermitian metric, symplectic form, area, cusp curves, weak convergence.

We will discuss the concepts and ideas about Gromov compactness theorem in this presentation. For example, the closed-surface version of GCT is

Theorem 1 (GCT)**.** *Let (M, J, μ) be an almost complex manifold under a hermitian metric. Let S be a closed surface and (j_n) a sequence of complex structures on S. Suppose*

$$f_n : (S, j_n) \to (M, J, \mu)$$

is a sequence of J-holomorphic curves of uniformly bounded areas. Then, there exists a subsequence of f_n converging weakly to a cusp curve $\overline{f} : \overline{S} \to M$.

The first part is about basic concepts appeared in the statement. (M, J, μ) is an almost complex manifold. We defines what a J-holomorphic curve,

$$u : (S, j) \to (M, J, \mu),$$

and its area mean. Then, singular Riemann surfaces and weak convergence will be introduced. In particular, a simple example about weak convergence will be described.

The second part is about some theorems on the theory of holomorphic curves. We have Schwartz lemma, monotonicity lemma, theorem of removable singularities, Weierstrass theorem, etc. They build up the ideas about holomorphic curves and resemble well-known results in complex analysis. We also use these results to formulate GCT on closed curves. Some results in hyperbolic geometry will be borrowed but not described in details. The essence is to replace convergence in maps by convergence in length, using double pant decomposition.

The third part is about GCT on compact curves. At this point ω has to be symplectic, and we introduce completely real submanifolds in (M, J). We go back to the Cauchy-Riemann equation at first place

$$\frac{\partial u}{\partial s} + J(u)\frac{\partial u}{\partial t} = 0.$$

Variational method provides a way to get GCT on compact curves. At the end, CR equation is a special case of Floer equation. We can understand this moduli space of holomorphic curves in a way similar to the construction of Floer homology.

Here are some references used in this presentation.

1. Gromov's Compactness Theorem for Pseude-holomorphic Curves, C Hummel.

2. J-holomorphic Curves and Symplectic Topology, D. McDuff & D. Salamon.

3. Gromov's Compactness Theorem for Pseudo Holomorphic Curves, R. Ye.

4. Morse Theory and Floer Homology, M. Andin & M. Diamian.