Chaos and Other Issues in Mathematical Finance

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Major developments in mathematical finance have come from the study of two deterministic parabolic partial differential equations, the Nobel Prize winning Black-Scholes equation for stock options

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2} x^2 \frac{\partial^2 u}{\partial x^2} + rx\frac{\partial u}{\partial x} - ru,$$

and the Cox-Ingersoll-Ross equation for zero coupon bonds

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2} x \frac{\partial^2 u}{\partial x^2} + (\beta x + \gamma)\frac{\partial u}{\partial x} - xu,$$

where $(x,t) \in (0,\infty) \times [0,\infty)$. Each has a particular initial condition

$$u(x,0) = u_0(x)$$

which is dictated by the economics of the problem.

We study these problems in weighted sup norm Banach spaces whose functions are unbounded near infinity (and possibly also near 0). The Black-Scholes equation has a closed formed solution (which we will derive) and is governed by a semigroup that is strongly continuous, quasicontractive, and chaotic. New extentions to time dependent coefficients will be given for this model and the resulting family of evolution operators are also chaotic.

The Cox-Ingersoll-Ross equation is governed by a strongly continuous quasicontractive semigroup. But the solution of this equation is far more complicated to describe; it is given by a new type of Feynman-Kac formula. Extensions to more general potential terms and time dependent variations of the model will be explained.