

ABSTRACT. The *indecisive infinite sum*

$$\sum_{n=1}^{\infty} (-1)^{n-1} = 1 - 1 + 1 - 1 + 1 - 1 + \dots$$

does not converge in the usual sense from Calculus to anything at all... Nevertheless, we will see that in a certain *averaging sense*, called *Cesàro convergence*, the *sequence of partial sums* of the above series $(S_N)_{N \in \mathbb{N}} = (1, 0, 1, 0, 1, 0, 1, 0, \dots)$ does converge: **to** $\frac{1}{2}$.

What about *infinite sums* that are even more indecisive than $\sum_{n=1}^{\infty} (-1)^{n-1}$:

$$\sum_{n=1}^{\infty} (-1)^{n-1} n \quad , \quad \sum_{n=0}^{\infty} (-1)^n 2^n \quad , \quad \text{or even} \quad \sum_{n=0}^{\infty} (-1)^n n! \quad \dots ?$$

These series fail to converge in a spectacular fashion, since their partial sums are *unbounded* and *oscillatory*. Yet we will see that we can still *persuade* them to *converge* using other methods, including those of *Borel*.