The Beautiful Math Behind Calculus 3; Spring 2017, Esmayli

We do integrate a function f defined on a surface S (a curved elastic sheet in space) by describing it as a function $f(x, y)$ or $f(x, z)$ or $f(y, z)$ defined on its shadow over one of the coordinate planes. By such projection, we obtain a function of two variables, defined on a planar region, whose integral to us then becomes meaningful and calculable. But...

Does the integral of a function f defined, say, on a torus really mean anything its own?

An independent object, regardless of how one calculates its number value?

Functions to us are introduced by $f(x)$, that is f of something. We are more comfortable with $f(x, y, z, t)$ than with $f(s)$, s being a point on a curved space because in the former we have something to plug into f. We get nervous in the latter case: Do I plug in a shape into f?! What does $f(3s)$ would then mean?! How could I possibly integrate such a creature? What is its anti-derivative even?

In this talk, I will put a face (and meaning) to the expression

$$\int_S f d\sigma.$$

You have seen this many times but viewed it as formal symbolic expression. You may never have questioned what each component of this might really stands for. Through many tangible examples I will discuss that $d\sigma$ **indeed does have a very deep geometric meaning**, which enables us to interpret that integral in an *intrinsic* way on any sphere.

We will have fun integrating functions on a donut, and similar shapes!

Only near the conclusion will I reveal that we have been discussing manifolds!

For an introduction to Manifolds in similar lines, see my posts on AMS Graduate Blog: http://blogs.ams.org/mathgradblog/author/besmayli