Mapped Finite Element Methods: higher order methods for simulating curvilinear crack propagation

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High order finite element solutions of crack (and re-entrant corner problems) in elasticity are notoriously challenging to attain. The slow convergence rates for these classes of problems are tied to the low degree of regularity possessed by the solution. The need for high order solutions is motivated by long running simulations of crack propagation. In fact, as the error in the crack path grows non-linearly, it is of uttermost importance to accurately resolve the elasticity fields at each step in time. The above ensures that the final computed crack paths are within reasonable tolerance of the exact one without excessive computational expenditures.

To this extent, we develop higher order finite element methods for crack (and re-entrant corner) problems in elasticity. The method [2] exploits the a priori knowledge of the singular behavior of the fields to construct an alternate regular solution. Solving for the alternate problem yields optimal rates of convergence and high order of accuracy. Namely, standard finite element error estimates in the L^2 and H^1 norm are proved to hold, both for the approximation of the alternate solution and the composed approximation of the alternate solution. The salient feature of the method is the lack of additional degrees of freedom in comparison with standard Galerkin finite element methods. Effectively, for the same computational cost we obtain a higher order of accuracy. Furthermore, unlike other state of the art tools, the method preserves the well conditioned nature of the system of equations and does not require the knowledge of the exact asymptotic behavior, hence the method can be readily applied to singular problems for which we are not endowed with an explicit asymptotic solution, such as cracks in graded materials. The method is verified with respect several analytical solutions in two- and three-dimensions accounting for the possible curvilinear (non-planar) geometry of cracks as well as material inhomogeneities.

We further motivate the need for higher order solutions by showcasing the rapidly convergent computation of the stress intensity factors through the use of interaction integrals [1] with the computed approximation of the elasticity fields.

The application of the method are showcased for complex fracturing problems, in particular for simulations of oscillatory fracture instabilities in thermoelastic materials subjected to large temperature gradients.

References

[1] Maurizio M. Chiaramonte, Yongxing Shen, Leon M. Keer, and Adrian J. Lew. Computing stress intensity factors for curvilinear cracks. *International Journal for Numerical Methods in Engineering*, 104(4):260–296, oct 2015.

[2] Maurizio M. Chiaramonte, Yongxing Shen, and Adrian J. Lew. Mapped finite element methods: High-order approximations of problems on domains with cracks and corners. *International Journal for Numerical Methods in Engineering*, 2017.