CHARACTERIZATIONS OF MEASURES IN THE DUAL OF BV AND RELATED ISOMETRIC ISOMORPHISMS

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Abstract. The solvability of the equation $\text{div}\boldsymbol{F} = \mu$ is connected to the analysis of BV^*, the dual of the space of functions of bounded variation. It is an open problem in geometric measure theory to characterize all the elements of BV^*. In this talk we present the characterization of all (signed) measures in $BV_{\frac{n}{n-1}}(\mathbb{R}^n)^*$, where $BV_{\frac{n}{n-1}}(\mathbb{R}^n)$ is defined as the space of all functions u in $L^{\frac{n}{n-1}}(\mathbb{R}^n)$ such that Du is a finite vector-valued measure. Moreover, we show that the measures in $BV_{\frac{n}{n-1}}(\mathbb{R}^n)^*$ coincide with the measures in $\dot{W}^{1,1}(\mathbb{R}^n)^*$. The space $\dot{W}^{1,1}(\mathbb{R}^n)^*$ is known as the G space in image processing and it plays a key role in modeling the noise of an image.

As a consequence of our characterizations, an old issue raised by Meyers and Ziemer is resolved by constructing a locally integrable function f such that f belongs to $BV(\mathbb{R}^n)^*$ but $|f|$ does not belong to $BV(\mathbb{R}^n)^*$ (we will show that $BV(\mathbb{R}^n)^*$ and $BV_{\frac{n}{n-1}}(\mathbb{R}^n)^*$ are isometrically isomorphic). For a bounded open set Ω with Lipschitz boundary, we characterize the measures in the dual space $BV_0(\Omega)^*$. We make precise the definition of $BV_0(\Omega)$, which is the space of functions of bounded variation with zero trace on the boundary of Ω. We show that the measures in $BV_0(\Omega)^*$ coincide with the measures in $W_0^{1,1}(\Omega)^*$.

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Date: September 7, 2017.
1991 *Mathematics Subject Classification.* 35A01, 46N20, 28A99, 26B30.