Helmholtz decomposition for (one-point) gradients of fractional order over bounded domains

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Abstract

The study of this talk aims to continue further with the nonlocal calculus developed for nonlocal gradients defined as

$$D_\delta^s u(x) = c_{n,s} \int_{B(x,\delta)} \frac{u(x)-u(y)}{|x-y|} \frac{x-y}{|x-y|} \frac{w_\delta(x-y)}{|x-y|^{n-1+s}} \, dy, \quad x \in \Omega$$

for $u : \Omega \cup B(0,\delta) \to \mathbb{R}$. This operator keeps a degree of fractional differentiability while providing a framework over bounded domains. Following some previous result regarding nonlocal versions of the Fundamental Theorem of Calculus, Poincaré Sobolev inequalities, Piola identity or compact embedding on the one hand and analogous of the Helmholtz decomposition (respectively for two-points nonlocal gradients, one-point gradient with integrable kernels or the Riesz fractional one) on the other hand, we would like to show a new nonlocal Helmholtz decomposition suitable in this framework over bounded domains over a strongly singular nonlocal gradient. One of the motivations of nonlocal models is that they may able to provide a framework for more general (or singular) phenomena, since they may not need to use classical derivatives.

The Helmholtz decomposition is a relevant result in mathematics a fluid mechanics which states that any (sufficiently smooth) vector field can be written as the sum of curl-free vector field plus a divergence-free one.