COUNTING G-BUNDLES OVER $\mathbb{P}^1$

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Recently Higgs bundles have received a lot of attention. A Higgs bundle over an algebraic curve is a vector bundle with a twisted endomorphism which roughly speaking can be thought of as a matrix of 1-forms on the curve. An important question is to calculate the volume of the groupoid of Higgs bundles over finite fields. In 2014, Olivier Schiffmann succeeded in finding the corresponding generating function and together with Mozvogoy reduced the problem to counting pairs of a vector bundle and a nilpotent endomorphism. It was generalized recently by Anton Mellit to the case of vector bundles with a nilpotent endomorphism preserving flags at marked points. An important part of Mellit's calculation is counting in the case of $\mathbb{P}^1$ and 2 marked points which allows him to relate the corresponding generating function with the Macdonald polynomials. In this talk, I will start by reviewing the notions of reductive groups, principal G-bundles and Mellit's result. I will then discuss the generalization to counting G-bundles with nilpotent sections to the case of $\mathbb{P}^1$ and two points for any reductive algebraic group G and show that we get an explicit formula in this case.