

COLLOQUIUM
UNIVERSITY OF PITTSBURGH
FRIDAY, JANUARY 22, 2016

704 THACKERAY HALL
3:30 P.M.

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DETERMINISTIC & PROBABILISTIC WELL-POSEDNESS
RESULTS FOR NONLINEAR SCHRÖDINGER
AND WAVE EQUATIONS

ABSTRACT: In this talk, we will discuss nonlinear Schrödinger and wave equations, which arise as mathematical models of a wide variety of physical phenomena. The equations also serve as important model problems in the broader study of PDE. We will give an overview of the treatment of these equations from both deterministic and probabilistic viewpoints. From the deterministic standpoint, a major open question is long-time existence and qualitative behavior of solutions when the nonlinearity is supercritical with respect to the conserved energy. Such questions are open even in the case of smooth compactly supported initial data with radial symmetry. Nevertheless, some partial results are possible, leading to a global well-posedness and scattering result for solutions which satisfy an a priori bound.

From the probabilistic perspective, we turn our attention to initial data which is supercritical with respect to the scaling of the nonlinearity. For this class of data, the initial value problems are ill-posed: uniform continuity of the solution map cannot hold, and we are therefore required to go beyond deterministic constructions. One resolution is to consider classes of randomized initial data, where the results are asked to hold almost surely in the randomization—that is, excluding a measure zero set of initial data. We will discuss recent results in this direction, partially obtained in collaboration with J. Bourgain. The main tools include a careful choice of function spaces, a priori bounds for the nonlinear evolutions and associated linearizations, and aspects of the algebraic structure arising from the Hamiltonian nature of the problems.

Refreshments served at 3:00 p.m.
in the Math Dept. COMMON ROOM, Thackeray 705

*The speaker is a candidate for a position in the Department.