COLLOQUIUM
UNIVERSITY OF PITTSBURGH
TUESDAY, JANUARY 12, 2016

704 THACKERAY HALL
3:00 P.M.

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MULTISCALE ANALYSIS OF RECTIFIABLE SETS

ABSTRACT: A set $E \subseteq \mathbb{R}^n$ is said to be d-rectifiable if it may be covered up to d-dimensional measure zero by Lipschitz images of $\mathbb{R}^d$. One can think of these as measure-theoretic analogues of differentiable manifolds: rather than requiring E to be locally parametrized by smooth chart-maps, we require E to be parametrized on sets of positive measure by Lipschitz maps. These are natural objects to work with as some results originally proven in smooth settings generalize to rectifiable sets, and occasionally this is the most general setting where they can hold. They arise in complex analysis, the study of the Dirichlet problem in domains with non-smooth boundaries, and the boundedness of singular integral operators on sets other than Euclidean space.

In some of these problems, knowing a set is rectifiable is not enough and more quantitative information about the multiscale behavior of the set is needed. A Lipschitz curve, for example, is differentiable almost everywhere, so we know that at almost every point the curve looks roughly affine or flat as we zoom in. A theorem of Peter Jones, however, quantifies how often such a curve is not approximately flat. In fact, he characterizes exactly when an arbitrary set may be contained in a curve of finite length in terms of a square sum measuring how flat the set is at each scale and location. This is called the Analyst's Traveling Salesman Theorem. In this expository talk, I will give an overview of the field of quantitative rectifiability, its applications, and some recent generalizations of the traveling salesman theorem.

Refreshments served at 2:30 p.m.
in the Math Dept. COMMON ROOM, Thackeray 705

*The speaker is a candidate for a position in the Department.