Kaveh's main areas of research are algebraic geometry and Lie theory. He is interested in combinatorial aspects of algebraic geometry and representation theory, and their connections with convex geometry. One of his current research interests is the theory of Newton-Okounkov bodies. This theory was initiated by seminal work of Andrei Okounkov (2006 Fields medalist).

Algebraic geometry is concerned with the study of algebraic varieties. An algebraic variety is the set of solutions of a number of polynomial equations in several variables (often considered over complex numbers). Toric varieties are a special class of varieties defined by binomial equations. In the past few decades, convex geometry and combinatorics of convex polytopes have played important parts in algebraic geometry, representation theory and symplectic geometry. This reaches its climax in the theory of toric varieties where there is a beautiful correspondence between the geometry of toric varieties and combinatorics of convex polytopes. The newly emerged theory of Newton-Okounkov bodies attempts to vastly generalize the correspondence between toric varieties and convex lattice polytopes. The Newton-Okounkov bodies are a far generalization of the (classical) notion of the Newton polytope of a (Laurent) polynomial in several variables.

As a simple example consider the Laurent polynomial $f(x, y) = a_1xy + a_2x^{-2}y + a_3xy^{-2}$, where the coefficients a_1, a_2, a_3 are nonzero complex numbers (Laurent polynomial just means that negative exponents are also allowed). The Laurent polynomial f has 3 monomials and the exponents (of the variables x, y) in these monomials can be represented as pairs of integers $(1, 1)$, $(-2, 1)$ and $(1, -2)$. The Newton polytope of f is the convex hull of these 3 points in $\mathbb{R}^2$ which is just a triangle Δ (shown in the figure below):

(Picture of the triangle goes here)

It turns out the the geometry and topology of the curve:

$$X(f) = \{(x, y) \in (\mathbb{C} \setminus 0)^2 \mid f(x, y) = 0\},$$

is closely related to the convex geometry of Δ. For example one shows that if $g(x, y) = b_1xy + b_2x^{-2}y + b_3xy^{-2}$ is another polynomial with the same set of exponents, then, provided that the coefficients a_i and b_i are general enough, the number of solutions of the system

$$\{(x, y) \in (\mathbb{C} \setminus 0)^2 \mid f(x, y) = g(x, y) = 0\},$$

is equal to 2 times the area of the triangle Δ, i.e. $2 \cdot (9/2) = 9$.

The main idea and construction in the theory of Newton-Okounkov bodies goes back to Andrei Okounkov in his influential works in the 90's and early 2000. He introduced, in passing, certain convex bodies which he used to prove the log-concavity of multiplicities of representations of reductive groups. Since 2008, Kaveh and A. G. Khovanskii developed, generalized and applied Okounkov's construction in a series of papers (also independently R. Lazarsfeld and M. Mustata). These works started the theory of Newton-Okounkov bodies (or Okounkov bodies for short). One of their papers appeared in Annals of Mathematics.

In a work which recently appeared in Inventiones Mathematicae, Kaveh uses the theory of Newton-Okounkov bodies to construct so-called *completely integrable systems* on a large class of smooth projective varieties. Smooth projective varieties are algebraic analogues of compact complex manifolds. This work uses the technique of deforming or *degenerating* a given projective variety to a toric variety. The figure below shows degenerating an elliptic curve (the donut shape) to a (pinched) sphere (simplest example of a toric variety). The loops with the arrows in the triangle show the flow of a circle action on the donut shape coming from this deformation.

(the picture Elliptic-curve.pdf goes here)

In another work also recently appeared in Duke Mathematical Journal, Kaveh connects the theory of Newton-Okounkov bodies with the representation theory of reductive algebraic groups (more specifically the theory of canonical/crystal bases of Kashiwara-Lusztig).