

MATH 0120 - BUSINESS CALCULUS

SAMPLE FINAL EXAM

1. Evaluate the following limits

(a) (5 points) $\lim_{x \rightarrow -1} \frac{3x^3 - 3x^2 - 6x}{x^2 + x}$

(b) (5 points) $\lim_{x \rightarrow 4} \frac{\sqrt{x} + 2}{x + 4}$

(c) (5 points) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

(d) (5 points) $\lim_{x \rightarrow 3^+} \frac{2}{3 - x}$

2. Find the derivatives of the following functions. **You do NOT need to simplify your answer.**

(a) (6 points) $f(x) = 3\sqrt{x}e^{7-x} + e^2$

(b) (6 points) $g(x) = \frac{x}{\ln(x^2 + 1)}$

(c) (6 points) $h(x) = \frac{x\sqrt[3]{x^4} - x + 2\sqrt{x} - 3}{x}$

3. Evaluate the following integrals.

(a) (8 points) $\int \frac{x\sqrt[3]{x^4} - x + 2\sqrt{x} - 3}{x} dx$

(b) (8 points) $\int \frac{e^{2x}}{3 + e^{2x}} dx$

(c) (8 points) $\int_0^1 2xe^{2x^2} dx$

(d) (8 points) $\int x^2 \ln x dx$

4. (10 points) Use **the limit definition** of the derivative to find the derivative of $f(x) = x^2 - 7x + 3$.
No CREDIT will be given if the limit definition is not used.

5. If total costs are given by $C(x) = 10x + 12$ and total revenues are given by $R(x) = 18x - x^2$, both in thousands of dollars, where x is the number of units.

(a) (3 points) Find the break-even points (break-even points are the numbers of units of production where a company's costs are equal to its revenue).

(b) (5 points) What is the maximum profit?

6. Follow the steps to graph the stated function.

$$f(x) = x^4 - 4x^3, \quad f'(x) = 4x^3 - 12x^2, \quad \text{and} \quad f''(x) = 12x^2 - 24x.$$

(a) (8 points) Make a sign diagram (or sign chart) for the first derivative of $f(x)$, and find all open intervals of increase and all open intervals of decrease.

- (b) (8 points) Make a sign diagram (or sign chart) for the second derivative of $f(x)$ and find all open intervals on which the graph is concave up and all open intervals on which the graph is concave down.
- (c) (6 points) Find the critical numbers and the inflection points of $f(x)$ and classify each critical point as a relative maximum, relative minimum, or inflection point.
- (d) (4 points) Sketch the graph of $y = f(x)$ by hand, plotting and labeling **only** the relative extreme points, inflection points and the y-intercept. To be considered correct, your graph must match your answers in parts a), b), and c).

7. (8 points) Find the equation of the line tangent to $x^4 + y^4 - 2x^2y^2 = 0$ at the point $(2, 1)$.

8. (8 points) Air is being pumped into a spherical balloon so that its volume increases at a rate of $100 \text{ cm}^3/\text{s}$. How fast is the radius of the balloon increasing when the diameter is 40 cm? **Indicate units in your answer.** Show all work. (NOTE: For a sphere, $V = \frac{4}{3}\pi r^3$).

9. Through some marketing research, Megadodo Publications knows that its best-selling book “The Hitchhikers Guide to the Galaxy” has the demand function $D(p) = 300 - p^2$ where p is the price of the book and D is the daily U.S. demand.

(a) (5 points) Find the Elasticity of Demand, $E(p)$, for this book.

(b) (3 points) If the current price of the book is \$5 determine if the demand is elastic, inelastic or unit-elastic.

(c) (2 points) In order to raise revenue, should Megadodo Publishing raise or lower the price?

(d) (4 points) How much should Megadodo Publications charge for each book if it wants to maximize revenue?

10. (6 points) State **but do not evaluate** the expression which gives the area bounded by the two curves.

$$f(x) = x^2 - 4$$

$$g(x) = 8 - 2x^2$$

11. (8 points) A company's marginal cost function is $MC(x) = 10e^{-0.01x}$ where x is the number of units. Supposing that fixed costs are \$500, find the cost function.

12. Suppose for a certain product the demand function is $d(x) = 600 - 10x^2$ and the supply function is $s(x) = 40x$.

(a) (4 points) What is the market demand for x ?

(b) (2 points) What is the market price for x ?

(c) (6 points) State **but do not evaluate** the expression that gives the consumer's surplus at the market demand.

(d) (6 points) State **but do not evaluate** the expression that gives the producer's surplus at the market demand.

13. (12 points) Find all critical points extreme of the function below and classify each as a relative maximum, relative minimum, or saddle point.

$$f(x, y) = x^2 + y^3 - 6x - 12y.$$

14. (12 points) A company manufactures two products with x = the number of units of product A produced and y = the number of units of product B produced. Because of limited materials and capital, the quantities produced must satisfy the equation $4x + 2y = 80$ (this is called a *production possibilities curve*). Given the company's profit function is $P = 4x^2 + y^2$, use Lagrange Multipliers to find the production levels of products A and B that maximize the company's profit.