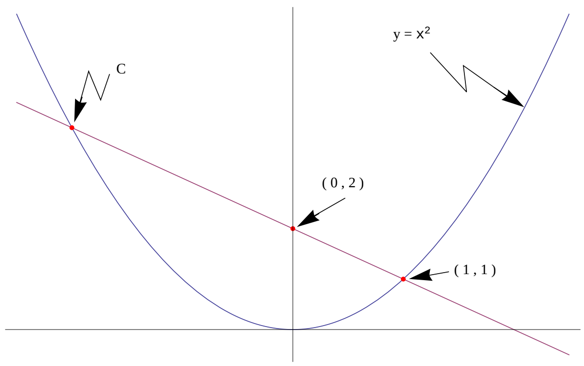
MATH 0120 - BUSINESS CALCULUS

SAMPLE FINAL EXAM

(1) (a) Using the graph below, find the coordinates of the point C. [10 points]



(b) Simplify the following expression as much as possible [10 points]:

$$\frac{\left(\dfrac{y^3-1}{y^2-1}\right)}{\left(\dfrac{y^2+y+1}{y^2+2y+1}\right)}$$

where $y \neq \pm 1$

(2) Find the following limits [12 points] :

(a) $\displaystyle\lim_{x\to -2}\left(\frac{x+2}{x^2-4}\right)$

(b) $\displaystyle\lim_{h\to 0}\left(\frac{(3+h)^2-9}{h}\right)$

(c) $\displaystyle\lim_{x\to 2^+}\left(\frac{(x+2)^2}{x^2-4}\right)$

(3) Using the **definition of the derivative only**, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$,[10 points]

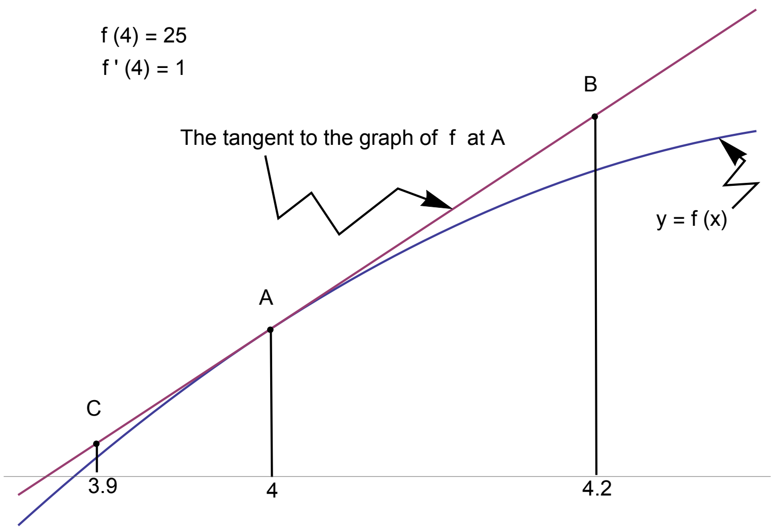
(a) find the derivative function of

$$f(x) = \frac{1}{x}.$$

(b) Using the derivative function you found in (a) above, find the equation of the tangent line to $f(x)$ above at the point $(2, \frac{1}{2})$. [2 points]

(4) (a) The quantity, q, of a certain skateboard sold depends on the selling price, p, so we write $q = f(p)$. You are given that $f(140) = 15,000$ and $f'(140) = -100$. What does $f(140) = 15,000$ and $f'(140) = -100$ tell you about the sale of the skateboards ? [8 points]

(b) State the coordinates of the point A. Calculate the coordinates of the points B and C using the figure below. Recall : $f'(x)$ can also be written as $\frac{dy}{dx}$. [12 points]



(5) (a) Find $\frac{dy}{dx}$ **(Do not simplify your answer).** [7 points]

$$y = \frac{e^{2x}}{1 + e^{-2x}}$$

(b) Find $f'(x)$ **(Do not simplify your answer) :** [7 points]

$$f(x) = (-9x^2 - 2)\sqrt[3]{9x^2 + 2}$$

(c) Find $\dfrac{dy}{dx}$ where x and y are related by

$$x^3 + 2xy^2 + y^3 = 1$$

[8 points]

(6) Consider the function $f(x) = 3x^4 - 4x^3 + 1$. [For your information : $f(\frac{2}{3}) = \frac{33}{81} = 0.4074$.] [20 points]

(a) Find the critical number(s) of $f(x)$ and the open intervals on which $f(x)$ is increasing and decreasing.

(b) Find the inflection point(s) of $f(x)$ and the open intervals on which $f(x)$ is concave up and concave down.

(c) Determine whether $f(x)$ at each critical point has a local maximum value, a local minimum value or neither.

(d) Sketch the graph of $f(x)$ by hand, indicating a scale on both axes and labeling all local extreme point(s) and point(s) of inflection.

(7) (a) A whitewater rafting company knows that at a price of \$80 for a half-day trip, they will attract 300 customers. For every \$5 decrease in price it is estimated that they will attract an additional 30 customers.
What price should the company charge and how many customers should they attract to maximize their revenue ? [10 points]

(b) Suppose the demand function for a certain product is given by

$$q = D(p) = 5000 - 3p^2$$

where p is the price in dollars.

(i) find the elasticity of demand, $E(p)$, at a price of \$30. Is the demand at this price elastic or inelastic ? [8 points]

(ii) Based on the elasticity found in (a), do you expect the price to maximize revenue to be above or below \$30 ? [2 points]

(8) Find the following integrals. Leave your answer in exact form [24 points]:

(a)

$$\int_{e}^{e^3} \left(\frac{t^2 - 1}{t} \right) dt$$

(b)

$$\int \frac{x^3}{(1 + x^4)^{1/3}} \, dx$$

(c)

$$\int \left(\frac{\ln x}{x^2} \right) dx$$

(9) (a) A company's marginal cost function is $C'(x) = 0.015x^2 - 2x + 80$ dollars where x is the number of units produced in one day. The company has a fixed cost of \$2000 per day.

(i) Find the total cost of producing x units per day. [7 points]

(ii) If the current production level is $x = 10$ units per day, determine the total change in cost if the production level is raised to $x = 20$ units per day.[5 points]

(b) Set up but **do not evaluate** the integral that gives the area of the region bounded by the two curves[10 points] :

$$y_1 = 2x^2 \qquad \text{and} \qquad y_2 = x^3 - 3x$$

(10) (a) A company sells two types of electric blenders, Model A and Model B, where x and y represent the number of units sold of Model A and Model B respectively. The daily revenue function is given by:

$$R(x, y) = -0.02x^2 + 80x - 0.05y^2 + 60y - 0.02xy$$

(i) Find $R_x(x, y)$ and $R_y(x, y)$. [8 points]

(ii) Calculate $R_x(100, 300)$ [3 points]

(iii) Calculate $R_y(100, 300)$ [3 points]

(b) For the function:

$$f(x, y) = 4y^3 + x^2 - 12y^2 - 36y + 2$$

Find all the critical point(s) of $f(x, y)$ and classify each as either a relative maximum, relative minimum or saddle point.[14 points]