Math 0120 - Business Calculus

Sample Final Exam

1. Evaluate the following limits

   (a) (5 points) $\lim_{x \to 4} \dfrac{\sqrt{x} - 1}{x + 2}$

   (b) (5 points) $\lim_{x \to 1} \dfrac{3x^3 + 3x^2 - 6x}{x^2 - x}$

   (c) (5 points) $\lim_{x \to 0^-} \dfrac{|x|}{x}$

   (d) (5 points) $\lim_{x \to -5^+} \dfrac{3}{x + 5}$

2. (10 points) Find the derivative of $f(x) = x^2 - 3x + 5$ using **the limit definition** of the derivative. NO CREDIT will be given if the limit definition is not used.

3. Find the derivatives. **You need not simplify**.

(a) (6 points) $f(x) = \frac{x}{\ln(2-x)}$

(b) (6 points) $g(x) = 2xe^{(3x+7)} + \pi$

(c) (6 points) $h(x) = (5x^2 - x)(x^4 + 4x)^{-2}$

4. (3 points) Let $R(x)$ represent the revenue, in dollars, generated by producing and selling x bicycles per day. Suppose it is known that $R(100) = -2$. If the company is currently producing 100 bicycles, would you recommend increasing production? Explain your answer.

5. (8 points) Suppose for a particular person that the time T (in minutes) required to learn a list of length n is

$$T = f(n) = 2n\sqrt{n-2}.$$

Find $f'(11)$ and interpret your answer.

6. Follow the steps to graph the stated function.

$$f(x) = -x^3 + 12x,\ f'(x) = -3x^2 + 12, \text{ and } f''(x) = -6x.$$

(a) (8 points) Make a sign diagram (or sign chart) for the first derivative of $f(x)$, and find all open intervals of increase and all open intervals of decrease.

(b) (8 points) Make a sign diagram (or sign chart) for the second derivative of $f(x)$ and find all open intervals on which the graph is concave up and all open intervals on which the graph is concave down.

(c) (6 points) Find the critical numbers and the inflection points of $f(x)$ and classify each critical point as a relative maximum, relative minimum, or inflection point.

(d) (4 points) Sketch the graph of $y = f(x)$ by hand, plotting and labeling **only** the relative extreme points, inflection points and the y-intercept. To be considered correct, your graph must match your answers in parts a), b), and c).

7. (8 points) Find the equation of the line tangent to $4x^2y - xy^3 = 0$ at the point $(1, 2)$.

8. (8 points) As a snowball is rolling down a hill its radius is increasing at a rate of 2 inches per second. Find the rate at which the volume is changing at the moment when the volume of the snowball is 36π cubic inches (NOTE: For a sphere, $V = \frac{4}{3}\pi r^3$).

9. A company's demand function for a luxury item is $D(p) = 180e^{-0.2p}$. The company is currently selling this luxury item for \$10 each.

   (a) (8 points) Find the Elasticity of Demand at this price and determine if the price is elastic, inelastic or unit-elastic.

   (b) (3 points) How much should the company charge for each luxury item it wants to maximize revenue?

10. (6 points) State **but do not evaluate** the expression which gives the area bounded by the two curves.

$$f(x) = 5 - x^2 \qquad\qquad g(x) = x^2 - 3$$

11. Evaluate.

(a) (8 points) $\int (\sqrt[3]{x^5} - e^{-2x} - \frac{3}{x^2} + 3)\, dx$

(b) (8 points) $\int_0^1 \frac{e^{2x} + 3}{e^{2x}} \; dx$

(c) (8 points) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \; dx$

(d) (8 points) $\int x \ln x \; dx$

12. (8 points) A company's marginal cost function is $MC(x) = 10e^{0.01x}$ where x is the number of units. Supposing that fixed costs are \$3000, find the cost function.

13. Suppose for a certain product the demand function is $d(x) = 600 - x^2$ and the supply function is $s(x) = 50x$.

    (a) (4 points) What is the market demand for x?

    (b) (2 points) What is the market price for x?

    (c) (6 points) State **but do not evaluate** the expression that gives the consumer's surplus at the market demand.

    (d) (6 points) State **but do not evaluate** the expression that gives the producer's surplus at the market demand.

14. (12 points) Find all critical points extreme of the function below and classify each as a relative maximum, relative minimum, or saddle point.

$$f(x, y) = y^3 - x^2 - 2x - 12y$$

15. (12 points) A company manufactures two products with $x =$ the number of units of product A produced and $y =$ the number of units of product B produced. Because of limited materials and capital, the quantities produced must satisfy the equation $4x + 2y = 80$ (this is called a *production possibilities curve*). Given the company's profit function is $P = 4x^2 + y^2$, use Lagrange Multipliers to find the production levels of products A and B that maximize the company's profit.