MATH 0120 - Business Calculus

SAMPLE FINAL EXAM

NAME (Print) _____ Student ID _____

NAME (Signature) _____

INSTRUCTIONS:

1. NO TABLES, BOOKS, NOTES, HEADPHONES, CALCULATORS, OR COMPUTERS MAY BE USED.
2. Show ALL of your calculations and display answers clearly. You may leave your final answers in exact form. Unjustified answers will receive no credit.
3. WRITE YOUR SOLUTIONS in the space provided. EXTRA SPACE is available on the BACKS of the pages. When using these back pages, clearly LABEL the problem, and also clearly indicate on the appropriate front page where your back-page solution (or continuation of a solution) is located.
4. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.
5. Academic Integrity Strictly Applies. Looking at another person's paper is reason to assume cheating and your paper will be taken.
6. All Cell phones and electronic devices must be OFF and put away and hats removed.

1. [10 points] Find an equation of the line that passes through the points $(-2, 3)$ and $(4, 6)$

2. Evaluate the limit

(a) [5 points] $\lim_{x \to 0} \dfrac{x^2 - 2x}{x^2 + 2x}$

(b) [5 points] $\lim_{x \to -3} \dfrac{x^2 - 9}{3 + x}$

3. Find the derivative of each function. You need not simplify the result.

(a) [5 points] $f(x) = 3\sqrt[3]{x^2} - \dfrac{2}{\sqrt{x}}$

(b) [5 points] $f(t) = \dfrac{t^2 - 1}{t + 2}$

(c) [5 points] $g(x) = x^2\sqrt{2 - x^2}$

(d) [10 points] $g(t) = e^{3t}\ln(1 - 2t)$

4. [10 points] Use the definition of derivative to find the derivative of $f(x) = x^2$.
(No credit will be given when the definition is not used).

5. (a) [10 points] Find $\dfrac{dy}{dx}$ if $x^3y - y^3 = 7$

(b) [5 points] Find the equation for the tangent line to the curve $x^3y - y^3 = 7$ at the point $(2, 1)$.

6. For the function $f(x) = \dfrac{8x}{x^2 + 4}$

(a) [10 points] find horizontal asymptotes

(b) [10 points] make a sign diagram for the derivative

(c) [5 points] find all relative maximum and minimum values

7. [10 points] Maximum Profit: A furniture store can sell 40 chairs per week at a price of \$80 each. The manager estimates that for each \$5 price reduction she can sell five more chairs per week. The chairs cost the store \$30 each. If x stands for the number of \$5 price reductions, find the price of the chairs and the quantity that maximize the profit. [Hint: Find the price $p(x)$ and the quantity sold $q(x)$ as functions of x. Revenue is price times quantity, cost is the cost per item times quantity, profit is revenue minus cost].

8. [10 points] Two cars start moving from the same point. One travels south at 40 mi/h and the other travels west at 30 mi/h. At what rate is the distance between the cars increasing two hours later?

9. [10 points] A bank offers 5% compounded continuously. How soon will a deposit triple? Leave answer in exact form.

10. [10 points] Find the area under the curve $y = xe^{x^2}$ when $0 \leq x \leq 2$. Leave answer in exact form.

11. For the demand function $d(x) = 30 - 0.1x^2$ and supply fuction $s(x) = 0.2x^2$ find

(a) [10 points] the market demand level (the positive value of x at which the demand function intersects the supply fuction).

(b) [10 points] the consumer's surplus at the market demand level found in part (a).

12. [10 points] The population of a town is increasing at the rate of $10t\,e^{t/2}$ people per year, where t is the number of years from now. Find the average gain in population during the next ten years. Leave your answer in exact form.

13. For the function $f(x, y) = e^{x+y} \ln x$

(a) [5 points] find the domain

(b) [10 points] find partials f_y and f_{yx}.

14. [10 points] If a company's profit function is

$P(x, y) = 2xy - 2x^2 - 3y^2 + 4x + 18y + 60$ thousand dollars,

find how many of each unit x and y should be produced in order to maximize the profit.

15. [10 points] Use Lagrange multipliers to find the maximum value of the function $f(x, y) = xy$ subject to the constraint $2x + y = 12$.

[Hint: Find CP. The maximum value of the function is attained at CP.]