Math 0120 Final Examination
Sample

Name (Print) _____ ID #_____

Signature _____

Instructor (circle one):

Lecture time (circle one):

## Instructions:

1. Show your University of Pittsburgh ID if requested.

2. Clearly print your name and PeopleSoft number and sign your name in the space above.  Circle the name of your lecturer and the time of your lecture.

3. Work each problem in the space provided. Extra space is available on the back of each exam sheet.  Clearly identify the problem for which the space is required when using the backs of sheets.

4. Show all calculations and display answers clearly.  Unjustified answers will receive no credit.

5. Write neatly and legibly.  Cross out any work that you do not wish to be considered for grading.

6. **No tables, books, notes, headphones, calculators, or computers may be used.  All derivatives and integrals are to be found by learned methods of calculus.**

### DO NOT WRITE BELOW THIS LINE

| Problem | Points | Score | Problem | Points | Score |
|---------|--------|-------|---------|--------|-------|
| 1 | 32 | | 6 | 20 | |
| 2 | 12 | | 7 | 40 | |
| 3 | 10 | | 8 | 20 | |
| 4 | 18 | | 9 | 15 | |
| 5 | 15 | | 10 | 18 | |
| | | | Total | 200 | |

1. (32 pts.) Find $f'(x)$. **You need not simplify**.

(a) $f(x) = e^{x^2} - \dfrac{1}{x} + e + \sqrt[4]{x^3}$

(b) $f(x) = (\dfrac{1}{x} + 3)^5$

(c) $f(x) = \dfrac{2x^3 - x}{1 - 2x}$

(d) $f(x) = x \ln(1 - x)$

2. (12 pts.) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$. Use this definition to find the derivative of $f(x) = \dfrac{1}{x^2}$.

3. (10 pts.) $x^2 y + xy^2 = 0$. Find $\dfrac{dy}{dx}$ at x=1, y = −1.

4. a. (8 pts.) If the national debt of a certain country (in billions of dollars) t years from now is given by

$N(t) = 4 + e^{0.01t}$, find the instantaneous rate of change and the relative rate of change at t = 0. Include units in your answers.

b. (10 pts.) Find an equation of the tangent line to $f(x) = x^2$ at x = 2.

5. (15 pts.) A cherry tree will yield 100 pounds of cherries now, which will sell for 80 cents a pound. Each week that the farmer waits will increase the yield by 10 pounds, but the selling price will decrease by 4 cents per pound. How long should the farmer wait to pick the fruit in order to maximize the revenue?

6. (20 pts.) Given $f(x) = x^3 - 12x$, do the following:

    (a) Make a sign diagram for the first derivative of f(x).

    (b) Make a sign diagram for the second derivative of f(x).

    (c) State the open intervals on which f(x) is increasing, decreasing, concave up and concave down.

    (d) Sketch the graph of y =f(x) by hand, labeling all relative extreme points and inflection points.

7. (40 pts.) Find the following integrals:

(a) $\int (5x^3 + \pi - \sqrt[3]{x^4}\, dx$

(b) $\int \frac{\sqrt{x}+1}{x}\, dx$

(c) $\int x\, e^{2x}\, dx$

(d) $\int (x^2 + 1)\sqrt{x^3 + 3x}\, dx$

8. (a) (10 pts.) Find the **area bounded by** the curves $y = x^3$ and $y = 4x$.

(b) (10 pts.) Find the **producers' surplus** for the supply function $s(x) = 0.04\,x$ at the demand level $x = 100$.

9. (15 pts.) Find all critical points of $f(x.y) = 6xy - x^3 - 3y^2$ and classify each as a relative maximum, relative minimum, or saddle point.

10. (18 pts.) Use the method of Lagrange multipliers to maximize **and** minimize $f(x,y) = 2x + y$ subject to the constraint $x^2 + 2y^2 = 72$. (Both extreme values exist.)