Math 0120 Final Examination
Sample

Name (Print) _____ **People Soft #**_____

Signature _____

Instructor (circle one):

Lecture time (circle one):

Instructions:

1. Show your University of Pittsburgh ID if requested.
2. Clearly print your name and PeopleSoft number and sign your name in the space above. Circle the name of your lecturer and the time of your lecture.
3. Work each problem in the space provided. Extra space is available on the back of each exam sheet. Clearly identify the problem for which the space is required when using the backs of sheets.
4. Show all calculations and display answers clearly. Unjustified answers will receive no credit.
5. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.
6. **No tables, books, notes, earphones, calculators, or computers may be used. All derivatives and integrals are to be found by methods of calculus learned in this course.**

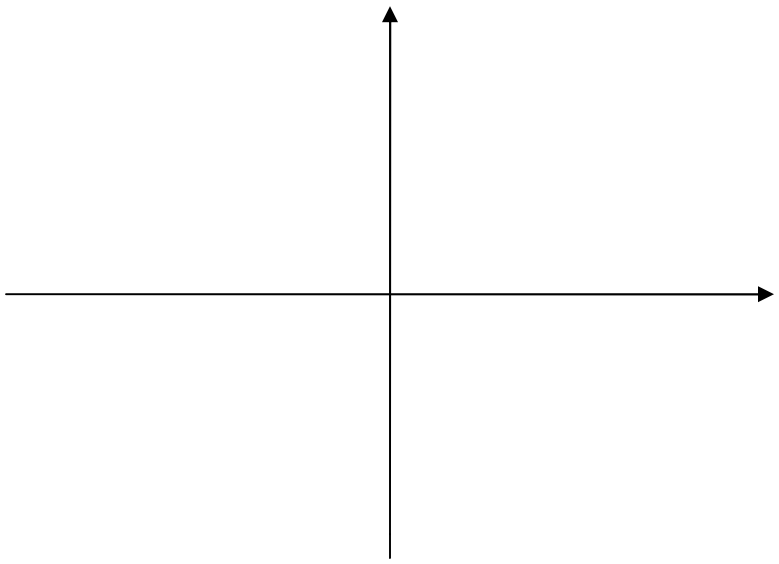
DO NOT WRITE BELOW THIS LINE

Problem	Points	Score	Problem	Points	Score
1	10		6	13	
2	17		7	27	
3	24		8	36	
4	25		9	13	
5	20		10	15	
			Total	200	

1. (a) (3 pts.) Find an equation of the line that passes through the points (1, 2) and (3,-2).

(b) (3 pts.) Find the vertex of the parabola given by $y = f(x) = 4x - x^2$.

(c) (4 pts.) Sketch the line and the parabola on the same set of axes, labeling the vertex of the parabola and the x- and y- intercepts of both.



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2. (a) (5 pts.) Find $\lim_{x \to 2} \frac{x-2}{x^3-8}$.

(b) (5 pts.) Let f(x) be a function. Write the definition of $f'(x)$, the derivative function.

(c) (7 pts.) Use this definition to find the derivative of f(x) = $\frac{1}{x-3}$.

3. (24 pts.) Find the derivatives. (**You need not simplify):**

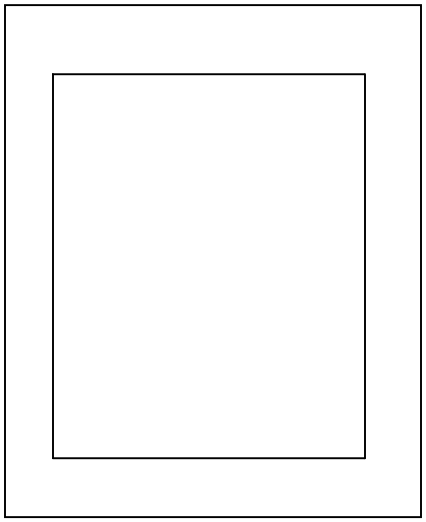
(a) $g(x) = \dfrac{x}{\ln(1-x)}$.

(b) $f(x) = (x^2 + x)(3x^2 - 2)^{-2}$.

(c) $x^3y - xy^3 = 0$. Find $\dfrac{dy}{dx}$.

4. (a) (12 pts.) An orchard contains 300 peach trees with each tree yielding 800 peaches. For every additional tree planted, the yield per tree decreases by 2 peaches. How many trees should be planted to maximize the total yield of the orchard?

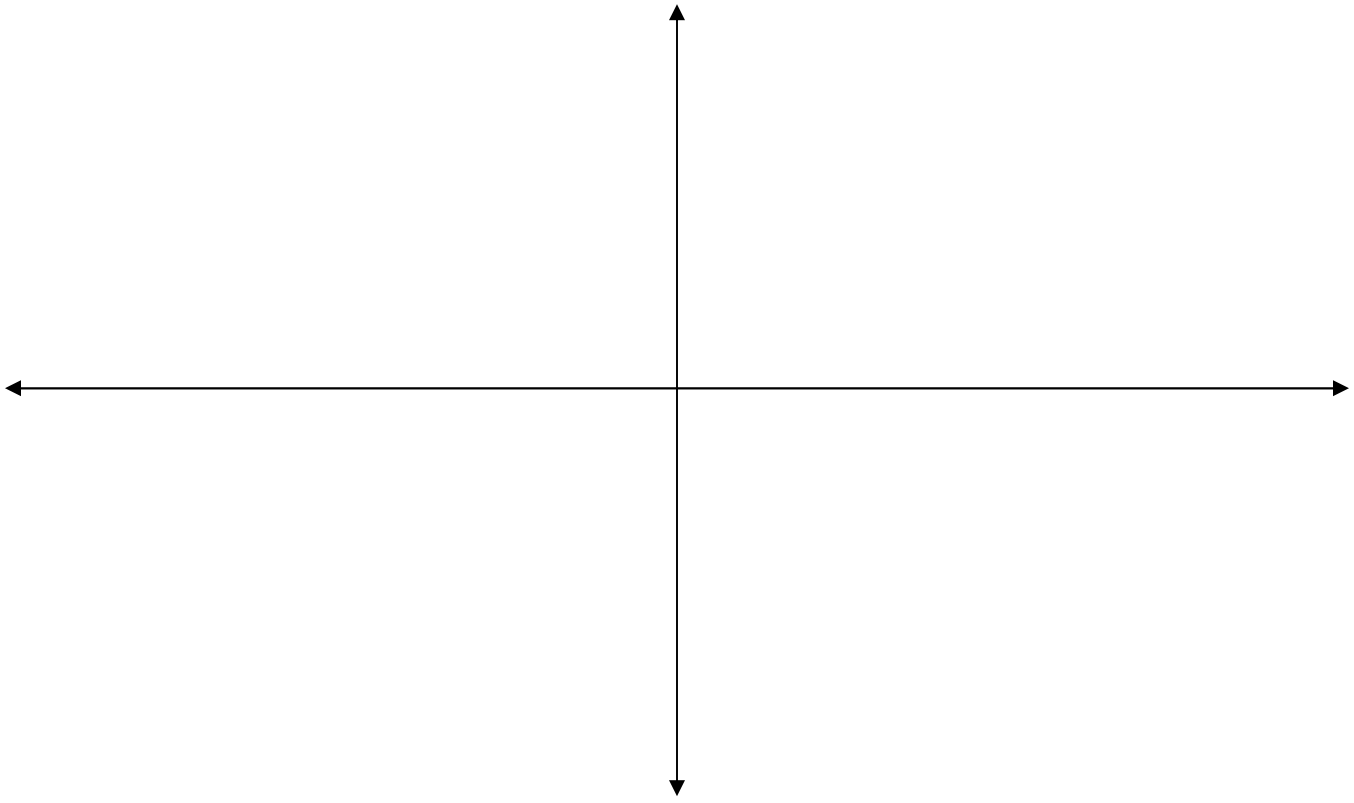
(b) (13 pts.) A poster is to have 2-inch margins at the top and bottom and 1½-inch margins on the sides. The total area is to be 300 square inches. Find the dimensions that will maximize the print area of the poster.



5. (20 pts.) $f(x) = x^4 + 4x^3 = x^3(x + 4)$, $f'(x) = 4x^3 + 12x^2 = 4x^2(x + 3)$, and $f''(x) = 12x^2 + 24x = 12x(x + 2)$.

Give a specific answer to each part:

(a) Construct sign charts for the first and second derivatives. (b) Find the critical numbers and the inflection points of f. (c) Find all open intervals of increase and decrease and open intervals on which the graph is concave up and concave down. (d) Classify each critical point as a relative maximum, relative minimum or neither. (e) Sketch the graph of y = f(x) by hand, plotting and labeling **only** the relative extreme points, inflection points and intercepts. **Use the factored form of f(x) to evaluate the functional values**.



6 (a) (7 pts.) $f(x) = 3 + e^{3-x}$. Find an equation of the tangent line at x = 3.

(b) (6 pts.) When a cold roast is placed into a 375° F oven, its temperatuture T, in degrees Fahrenheit at time t hours is $T(t) = 375 - 325e^{-.2t}$. Find the instantaneous and relative rates of change of T at the time the roast is placed in the oven (t = 0). **Include proper units in your answers**.

7. (a) (6 pts.) State the Fundamental Theorem of Calculus.

(b) (7 pts.) Find f(x) given that $f'(x) = e + e^x + \frac{1}{x}$ and f(1) = 1 + 2e.

(c) (14 pts.) Find the **area between** the curves **$f(x) = x^2 - x$** and **$g(x) = x - x^2$** on [-1,3].

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8. (36 pts.) Find the following integrals:

(a) $\int (\sqrt[3]{x^5} - e^{-2x} - \frac{3}{x^2} + 3)\, dx$

(b) $\int x \ln x \, dx$

(c) $\int \frac{e^{\frac{1}{x}}}{x^2} dx$

(d) $\int \frac{\sqrt{x} + 3}{x} dx$

9. (13 pts.) Find all critical point(s) of $\mathbf{f(x, y) = 2x^3 + 2y^3 - 12xy + 5}$ and classify each as a relative maximum, relative minimum, or saddle point.

10. (15 pts.) Use the method of Lagrange multipliers to maximize and minimize $\mathbf{f(x, y) = 2x + y}$ subject to the constraint $\mathbf{x^2 + 2y^2 = 72}$. (Both extreme values exist).