Precalculus Math 200

Sample Final Exam

November 28, 2015

Q1.[pts] Solve the following equation.

$$| -2x + 1| - 5 = 10$$

Q2.[pts] Find the center, the foci, the length of the major axis , and the length of the minor axis for the ellipse defined by the equation

$$x^2 + 4x + 25y^2 - 50y = -4$$

Q3.[pts] Find an equation for the line passing through the point $(3, 2)$ and parallel to the line $y = -x - 16$.

Q4.[pts] Find the vertex of the parabola $y = -3x^2 - 6x + 9$ and determine whether it is a minimum or maximum .

Q5.[pts] Find all numbers x such that the following equation holds.

$$\log_3(8x - 7) = 2$$

Q6.[pts] Solve these equations for all possible real values of x.

(i) $e^{2x} + 2 = 8 - e^x$

(ii) $3^x = 27$

(iii) $4\log_3(5x - 3) = 12$

(iv) $\log_3(4x) - \log_3(x - 5) = 2$

Q7.[pts]   Construct an example of a rational function whose graph in the xy-plane has as vertical asymptotes the lines $x = 3$ and $x = 5$ and as an oblique asymptote the line $y = 2x - 3$.

Q8.[pts] If $15,000 is deposited in a saving account with an annual interest rate of 7% compounded continuously, how long will it take for this amount to double?

Q9.[pts] On a twelve hour clock, find the angle in degrees between the hour hand and the minute hand at 2 : 27. Your answer should be betweeen $0°$ and $180°$.

Q10.[pts] Suppose $\frac{3\pi}{2} < \theta < 2\pi$ and $\sin\theta = -0.6$. Evaluate the values of the other five trigonometric functions at θ.

Q11.[pts]   Suppose a 40-foot ladder is leaning against a a wall, making a 45°
angle with the ground (measured from a perpendicular line from the base of
the ladder to the wall).

(1) How high up the wall is the end of the ladder ?

(2) How far from the wall is the base of the ladder ?

Q12.[pts]   Find the exact values for the indicated quantities if we are given that:

$$\cos \frac{\pi}{12} = \frac{\sqrt{2 + \sqrt{3}}}{2} \quad \text{and} \quad \sin \frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

(i) $\cos \left(\frac{\pi}{24}\right)$ (ii) $\sin \left(\frac{\pi}{24}\right)$

(iii) $\cos \left(\frac{\pi}{16}\right)$ (iv) $\sin \left(\frac{\pi}{16}\right)$

v) $\tan \left(\frac{\pi}{24}\right)$ (vi) $\tan \left(\frac{\pi}{16}\right)$

(vii) $\sin \left(\frac{\pi}{24} + \frac{\pi}{16}\right)$ (viii) $\cos \left(\frac{\pi}{24} + \frac{\pi}{16}\right)$

ix) $\sin \left(\frac{\pi}{16} - \frac{\pi}{24}\right)$ (x) $\cos \left(\frac{\pi}{16} - \frac{\pi}{24}\right)$

xi) $\tan \left(\frac{\pi}{16} + \frac{\pi}{24}\right)$ (xii) $\tan \left(\frac{\pi}{16} - \frac{\pi}{24}\right)$

Q13.[pts]   Evaluate the following expressions

(i) $\cos^{-1}\cos(3\pi)$ (ii) $\sin^{-1}\sin\left(\frac{6\pi}{7}\right)$

(iii) $\cos^{-1}\cos(40°)$ (iv) $\sin^{-1}\sin(70°)$

(v) $\sin\left(\cos^{-1}\frac{1}{3}\right)$ (vi) $\cos\left(\sin^{-1}\frac{1}{4}\right)$

(vii) $\cos\left(\tan^{-1}(-4)\right)$ (vi) $\sin\left(-\sin^{-1}\frac{3}{13}\right)$
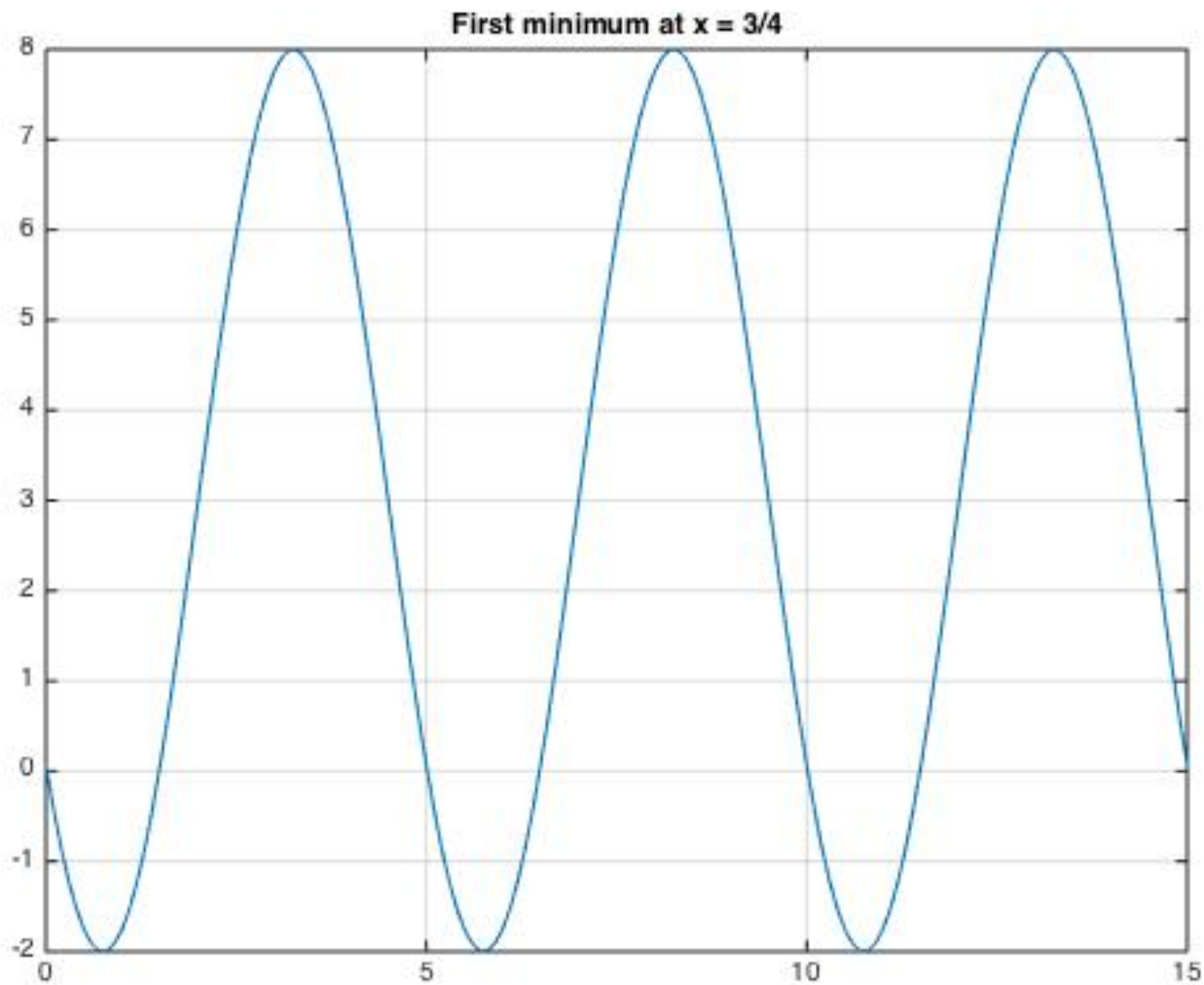
Q14.[6 pts]   Suppose t is such that $\cos^{-1}t = \frac{\pi}{17}$ Evaluate the following expressions :

(i) $\cos^{-1}(-t)$ (ii) $\sin^{-1}(t)$ (iii) $\sin^{-1}(-t)$

Q15.[pts] Find a function that models the periodic behavior described by the graph below. Yon need to use the fact that the first minimum occurs at $x = \frac{3}{4}$.

First minimum at x = 3/4

Q16.[pts]   Suppose the wind at airplane heights is 40 miles per hour (relative to the ground) moving 15° North of East. Relative to the wind, an airplane is flying at 450 miles per hour 20° South of the wind. Find the speed and direction of the airplane relative to the ground.

Q17.[pts]   Find a number t such that the vectors $\vec{u} = \langle 2\cos t, 4 \rangle$ and $\vec{v} = \langle 10, 3 \rangle$ are perpendicular.

Q18.[pts] Perform the indicated operation and write each of the following expressions in the form $a + bi$, where a and b are real numbers.

1 $(5 + 7i)(4 + 6i) =$

2 $\dfrac{4+3i}{5-2i} =$

3 $(\sqrt{11} + \sqrt{3}i)^2 =$

4 $\overline{(\sqrt{7} + \sqrt{3}\,i)} =$