MATH 1550: VECTOR ANALYSIS AND APPLICATIONS FALL 2025

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1. Where?

We shall meet on Tu Th from 4:00 p.m. to 5:15 p.m. at 703 Thackeray Hall.

Office Hours: Tu Th from 2:30 p.m. to 3:20 p.m., and by appointment.

2. What?

In this course we will discuss the mathematical tools needed to understand Field Theory. Many natural phenomena (electromagnetism, gravity, relativity, fluid flow, etc.) are described in terms of vector fields. At the end of this course you should be able to understand the Maxwell equations, the Navier-Stokes equations, and various conservation laws from mechanics.

We will consider extensions of the Fundamental Theorem of Calculus to higher dimensions: Green Theorem, the Divergence Theorem, and Stokes Theorem. We will also introduce the more advance language of Tensor Analysis, appropriate to study fields of matrices and other higher dimensional objects.

3. Topics

(1) Review of Math 0240 with emphasis on Green, Stokes, and Divergence theorems.
(2) Applications: Frenet-Serret formulas, Kepler Laws.
(3) Change of variables formula (changing coordinates), orthogonal curvilinear systems, Green, Stokes, and Divergence theorems for orthogonal curvilinear systems.
(4) The algebra of tensors. The derivative of a tensor. Proofs of vector identities using tensors.
(5) The metric tensor of a surface. Application: Riemmanian geometry of a surface: First and Second fundamental forms, Gauss curvature.
(6) Tensors in classical mechanics: Newton equations, continuity equations, flow map, equations of fluid mechanics (Maxwell, Euler, Navier-Stokes).
(7) Special relativity, invariance of the Maxwell Equations under Lorentz transformations. The wave equation.

Date: August 20, 2025.

Course Prerequisites: Math 0240 (Calculus III) or equivalent, and Math 1180 (Linear Algebra) or equivalent. As you know Mathematics is a very hierarchical discipline. While we will review basics concepts from these courses, you are expected to be knowledgeable of their content. Math 0420 is highly recommended.

4. Textbook

We do not have a required textbook.

- For an elementary classical treatment of tensors see [SPS], the Schaum's outline on Vector Analysis. The price is about $20 new at the usual sources for books. This textbook has hundreds of problems (480 to be exact) fully solved. It is great for the computational part of our course.
- Note however that In this course we aspire to understand not only **how** to do things, but also **why** things are the way they are. For the latter, I strongly recommend the classic [F]. Chapters 11 (Volume 1) and Chapters 2 and 3 (Volume II) are good places to start.
- A good advanced introduction to tensor calculus is [DK], another affordable book in the Schaum's outline series.
- More topics on tensor analysis can be found in [BT]. More topics on vector analysis can be found in [DS].

I will based my lectures in [DK] and [SPS]

5. What is expected of you?

There will be plenty of **homework assignments**. A ssignments completed in LaTeX, which is the standard mathematical typesetting package for science and technology will received a 5% bonus. LaTeX[1] is free and run in all computers. You could either download it to your computer, or use the free web version *Overleaf*[2]. If you don't use LaTeX, make sure your work can be read properly on a computer screen before you submit it. Please work on the problems assigned as hard as you can. This is very much a hands-on course

There will be 9 assignments (approximately one per week), a midterm examination and a **comprehensive final exam** at a date and place to be announced by the University Registrar.

Your final grade will be calculated as follows:

$$.40\,\text{Homework} + .25\text{Midterm} + .35\text{Final}$$

where each of "Homework", "Midterm" and "Final" has a maximum value of 100.

[1] `https://www.latex-project.org`

[2] `https://www.overleaf.com/`

Detailed academic policies and additional information about the course are at the course **Canvas page**[3], which you should check often.

References

[DK] Kay, David C., *Tensor Calculus*, Schaum's Outlines, McGraw Hill, 2011.

[F] Feynman, Richard P., *The Feynman Lectures in Physics*, Addison-Wesley and California Institute of Technology, 1963. Available on line at `https://www.feynmanlectures.caltech.edu`.

[SPS] Spiegel, Murray; Lipschutz, Seymour; and Spellman Dennis, *Vector Analysis*, Second Edition, Schaum's Outlines, McGraw Hill, 2009.

[BT] A. I. Borisenko, I. E. Tarapov, *Vector and Tensor Analysis with Applications*, Dover Publications, New York, 1979.

[DS] H. F. Davis, A. D. Snider, *Introduction to Vector Analysis*, 7th ed, William C Brown Pub; ISBN-13: 978-0697160997, ISBN-10: 0697160998.

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[3] `https://canvas.pitt.edu/courses/334709`