Syllabus for the PhD Preliminary Examination in Linear Algebra

Topics

Vector spaces: subspaces, linear independence, bases, dimension, isomorphism, linear functionals, dual space, adjoints, inverses and reducibility

Matrices and linear transformations: range, kernel, determinants, isomorphisms, change of basis, eigenvalues, eigenvectors, minimax

Theory of eigenvalues, Gersgorin discs, minimal polynomial, Cayley-Hamilton theorem, similarity, polar and singular value decomposition, spectral theorem, Jordan cannonical forms.

Hermitian, symmetric, and positive definite matricies. Matrix and vector norms, convergence of sequences, powers etc. of matrices

Inner product spaces: inner products, norms, orthogonality, projections, orthogonal complement, orthonormal basis, Gram-Schmidt orthogonalization, linear functionals, isometries, normal operators, spectral theory, basic inequalities such as Cauchy Schwarz

REFERENCES

P.R. Halmos, Finite Dimensional Vector Spaces, Springer, 1993

R. Horn and C. Johnson, Matrix Analysis, Cambridge, 1999

Peter Lax, Linear Algebra, Wiley-Interscience, 1997.

K. Hoffman and R. Kunze, Linear Algebra, 2nd ed., Prentice Hall, 1972

P. N. de Souza and J.-G. Silva, Berkeley Problems in Mathematics, Springer, Berlin, 2004, (Chapter 7: Linear Algebra)