Math 0280 - Linear Algebra - Sample Final

1. (7 points) Find the distance from the point $(1, 2, 3)$ to the line $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

2. (7 points) Determine whether the following vectors are linearly dependent or independent. If they are linearly dependent please find a linear dependence relationship among the vectors. Justify your answer.

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \text{ and } \ \vec{v}_3 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}.$$

3. (8 points) Consider the following linear system.

$$\begin{cases} x + y + kz &= 1 \\ x + ky + z &= 1 \\ kx + y + z &= -2 \end{cases}$$

Find all values of k such that the above linear system has

(i) no solution, (ii) a unique solution, and (iii) infinitely many solutions.

4. (8 points) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

or explain why it does not exist.

5. (20 points) For the matrix

$$A = \begin{bmatrix} 2 & -4 & 0 & 2 & 1 \\ 1 & -2 & -1 & -2 & -2 \\ -1 & 2 & 1 & 2 & 2 \\ 1 & -2 & 1 & 4 & 4 \end{bmatrix}$$

Find each of the following, Show all work.

(i) a basis for row(A); ii) a basis for col(A); (iii) a basis for null(A);

(iv) rank(A); (v) nullity(A).

6. (7 points) Find the standard matrix of the linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$, that reflects a vector about the $x-$axis, then rotates it counterclockwise by $60°$ about the origin and then reflects a vector about the $y-$axis.

7. (8 points) Use **Cramer's Rule** to solve the following linear system.

$$\begin{cases} x + y = 3 \\ 2x - 3y = 1. \end{cases}$$

NO CREDIT will be given, if **any other method** is used.

8. (20 points) The matrix A is given by

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Determine if A is diagonalizable or not. If it is, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$. (You DO NOT have to find P^{-1}).
If it is not diagonalizable, explain why.

9. (15 points) Suppose W is a subspace of $\mathbb{R}^5$, such that $W = \text{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

a) Use the Gram-Schmidt process to construct an orthogonal basis of W.

b) Find the orthogonal projection $\text{proj}_W(\vec{v})$ of vector $\vec{v} = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 2 \end{bmatrix}$ onto W.