MATH 0280 Final Examination, Sample 5 - ANSWERS

Problem 1. a) (the correct answers are not unique):

Possible choice of basis for $col(A)$: $[1,-1,0],\ [1,-2,2],\ [-2,0,5]$ (vertical vectors are listed horizontally to save space);

Possible choice of basis for $row(A)$: $[1,1,0,2],\ [0,-1,3,-2], [0,0,0,1]$ (or $[1,1,0,2],\ [-1,-2,3,0], [0,2,-6,5]$.)

b) $rank(A) = 3,\ nullity(A) = 1$.

c) Yes.

Problem 2.

a) $S = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

b) $S \circ T = \begin{pmatrix} \sqrt{2} & 0 & -2\sqrt{2} \\ \\ 0 & 2\sqrt{2} & 2\sqrt{2} \end{pmatrix},\ S \circ T(v) = \begin{bmatrix} -5\sqrt{2} \\ \\ 10\sqrt{2} \end{bmatrix}$

Problem 3. $\begin{pmatrix} -5 & 6 & 3 \\ 1 & -1 & -1 \\ 2 & -2 & -1 \end{pmatrix}$

Problem 4. $det(A) = -8,\ det(B) = -4,\ det(B^2A^{-1}) = -2$.

Problem 5. Diagonalizable (there are three distinct eigenvalues).

$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix},\ P = \begin{pmatrix} -3 & 0 & 1 \\ \\ \frac{1}{2} & 1 & \frac{3}{2} \\ \\ 1 & 0 & 1 \end{pmatrix}$

Problem 6. (vertical vectors are listed horizontally to save space:)

a) $[1,0,-2,1],\ [3,1,2,1],\ [2,-4,0,-2]$

b) $[3/5, 6/5, -3/5, 6/5]$.