MATH 0280 Final Examination, Sample 5

There are 6 problems for a total of 100 points in this exam.

(1) (15 pts) Consider the matrix below:

$$A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ -1 & -2 & 3 & 0 \\ 0 & 2 & -6 & 5 \end{bmatrix}$$

(a) Find bases for row(A) and col(A).
(b) Compute rank(A) and nullity(A).
(c) Determine whether $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ is in col(A).

(2) (15 pts)
(a) Write the standard matrix of the linear transformation $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by counter-clockwise rotation about the origin by an angle of $\pi/4$ (a.k.a. 45 degrees).
(b) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation with standard matrix below:

$$[T] = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 2 & 4 \end{bmatrix}$$

Compute the standard matrix of $S \circ T$, then compute $(S \circ T)(\mathbf{v})$, where $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Simplify your answer as much as possible.

(3) (15 pts) Find the inverse of the matrix below:

$$\begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 2 \\ 0 & -2 & 1 \end{bmatrix}$$

(4) (15 pts) Consider the matrices below:

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix}$$

Compute $\det A$, $\det B$, and $\det B^2 A^{-1}$.

(5) (20 pts) Determine whether the matrix A below is diagonalizable; and if so, find an invertible matrix P and diagonal matrix D such that $P^{-1}AP = D$.

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

(6) (20 pts) Consider the subspace V and vector $\mathbf{u}$ below:

$$V = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 4 \\ -2 \end{bmatrix} \right\} \qquad \mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

(a) Use the Gram-Schmidt process to find an orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for V.
(b) Compute the projection of $\mathbf{u}$ onto V.