MATH 0280 Final Examination, Sample 4-ANSWERS

Problem 1. A has eigenvalues $\lambda = 0$ with alg. and geom. multiplicities 2, and $\lambda = 1$ with alg. and geom. multiplicities 1.

$$A^{2012} = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ -8 & -4 & -4 \end{pmatrix}$$

Problem 2. a) $A = \begin{pmatrix} -2 & 2\sqrt{3} \\ & \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$

b) $A^{-1} = \begin{pmatrix} -\frac{1}{8} & -\frac{\sqrt{3}}{2} \\ & \\ \frac{\sqrt{3}}{8} & -\frac{1}{2} \end{pmatrix}.$

Problem 3. a) $A^{-1} = \begin{pmatrix} 1/3 & 1/2 & -1/6 \\ -4/9 & -1/6 & 7/18 \\ 2/9 & -1/6 & 1/18 \end{pmatrix}$

b)

$$\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 2 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 18 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

NOTE: This representation is not unique.

Problem 4. $\frac{1}{81}$.

Problem 5. Possible choice of a basis for $row(A) : [-1, 3, 4, 0, -2],\ [0, 1, -3, -1, 2]$.

Possible choice of a basis for $col(A)$: (column vectors are written horizontally here and below to save space): $[-1, 0, -3],\ [3, 1, 7]$.

Possible choice of a basis of $null(A)$: $[13, 3, 1, 0, 0],\ [3, 1, 0, 1, 0],\ [-8, -2, 0, 0, 1]$.
$rk(A) = dim(row(A)) = dim(col(A)) = 2$,
$nullity(A) = dim(null(A)) = 3$.

Problem 6. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -11 \\ -4 \\ -16 \end{bmatrix}$.

Problem 7. The answer is not unique. One of the possible choices:
$[1, 1, 1, 1],\ [1, -1, 0, 0],\ [-1, -1, 3, -1],\ [1, 1, 0, -2]$.
(When checking your answers, make sure that you have 4 vectors, such that every two of then are orthogonal to each other).