MATH 0280 Final Examination, Sample 4

Problem 1. Show that A is diagonalizable. Then find A^{2012}.

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ -8 & -4 & -4 \end{pmatrix}$$

Problem 2. a) Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, if T rotates a vector clockwise about the origin by $\frac{2\pi}{3}$, and then stretches it by a factor of 4 in the $x-$component.

b) Find the standard matrix of the inverse transformation T^{-1}, or show that the transformation T is not invertible.

Problem 3. a) Determine whether the given matrix A is invertible. If it is invertible, find its inverse:

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 2 & 3 & 3 \end{pmatrix}$$

b) Represent the matrix A as a product of elementary matrices or show that it is not possible.

Problem 4. Find $det(\frac{1}{3}A)$, if $A = \begin{pmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & -3 & 0 & 4 & 10 \end{pmatrix}$

Problem 5. Find bases for $row(A)$, $col(A)$ and $null(A)$, and the rank and nullity of A, if

$$A = \begin{pmatrix} -1 & 3 & 4 & 0 & -2 \\ 0 & 1 & -3 & -1 & 2 \\ -3 & 7 & 18 & 2 & -10 \end{pmatrix}$$

Problem 6. Solve the system of linear equations by using Cramer's method. Any other method will receive NO CREDIT.

$$\begin{cases} x - z &= \ \ 5 \\ 3y - x &= -1 \\ z - 2x + y &= \ \ 2 \end{cases}$$

Problem 7. Find an orthogonal basis of $\mathbb{R}^4$, containing vector $[1, 1, 1, 1]$.