

MATH 0280 Final Examination, Sample 3-ANSWERS

Problem 1. a) $\lambda = 0$ (alg. mult. 1); $\lambda = 1$ (alg. mult. 2)

b) For $\lambda = 0$: Basis of E_λ : $[1/2, 1/2, 1]$ (vertical vectors are written horizontally here and below to save space)

For $\lambda = 1$: Basis of E_λ : $\{[2, 1, 0], [-1, 0, 1]\}$

c) A is diagonalizable.

$$P = \begin{pmatrix} 1/2 & 2 & -1 \\ 1/2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Problem 2.

$$\text{a) } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -67 \\ 12 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -12 \\ 2 \\ 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} 28 \\ -5 \\ 1 \\ 0 \end{bmatrix} s$$

b) Yes. The system $(A|b)$, where A has column vectors v_1, v_2, v_3, v_4 is consistent (from part a).

$$\text{Problem 3. a) } \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 1 \end{bmatrix}.$$

$$\text{b) } \begin{bmatrix} -31/70 \\ -11/35 \\ 31/70 \\ -26/35 \end{bmatrix}.$$

$$\text{c) } \begin{bmatrix} -3/2 \\ -1/4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Problem 4. 6.

$$\text{Problem 5. } \begin{pmatrix} 0 & 0 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$$

Problem 6.

a) Basis for $\text{row}(A)$: $[1, 3, -4, 0, 0]$, $[0, 1, -2, -1, -3]$, $[0, 0, 0, 2, 7]$.

Basis for $\text{col}(A)$: $[2, 0, 4, 3]$, $[1, 2, 3, 4]$, $[1, 0, 1, 1]$. (Here and below vertical vectors are written horizontally to save space)

b) $[-2, 2, 1, 0, 0]$, $[3/2, -1/2, 0, -7/2, 1]$

c) $\text{rank}(A) = 3$, $\text{nullity}(A) = 2$.

Problem 7. $A^{-1} = \begin{pmatrix} -1 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1/2 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$