MATH 0280 Final Examination, Sample 3

Problem 1.(30 pts) Matrix A is given as

$$A = \begin{bmatrix} 0 & 2 & -1 \\ -1 & 3 & -1 \\ -2 & 4 & -1 \end{bmatrix}$$

a)(10 pts) Find all eigenvalues of A.

b)(10 pts) Find a basis for each eigenspace of A.

c)(10 pts) Determine whether A is diagonalizable. If it is, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

NOTE: You do not have to find P^{-1}.

Problem 2.(30 pts)

a)(20 pts) Solve the following system of linear equations.

$$\begin{cases} 2x_1 + 11x_2 - x_3 + 2x_4 &= -2 \\ x_1 + 6x_2 + 2x_3 &= 5 \\ x_1 + 7x_2 + 7x_3 - 2x_4 &= 17 \end{cases}$$

b)(10 pts) Determine whether the vector $\begin{bmatrix} -2 \\ 5 \\ 17 \end{bmatrix}$ is a linear combination of the vectors $\mathbf{v_1} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 11 \\ 6 \\ 7 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}$, and $\mathbf{v_4} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$.

Explain in details your conclusion.

Problem 3.(30 pts) The subspace W is defined as $W = \text{span}\{\mathbf{x_1}, \mathbf{x_2}\}$, where

$$\mathbf{x_1} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix} \text{ and } \mathbf{x_2} = \begin{bmatrix} 2 \\ 0 \\ -2 \\ 3 \end{bmatrix}.$$

a)(10 pts) Use Gram-Schmidt Process to find an orthogonal basis of W.

b)(10 pts) Find the orthogonal projection onto W of the vector $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$.

c)(10 pts) Find a basis of the orthogonal complement of W.

Problem 4.(25 pts) Evaluate the determinant of the matrix C.

$$C = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 2 & 3 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 2 & 1 & 1 & 2 \end{bmatrix}$$

Problem 5.(25 pts) Find the standard matrix of the linear transformation from R^2 to R^2, obtained by the rotation by $\frac{\pi}{3}$ in the counterclockwise direction followed by the orthogonal projection onto the y-axis.

Problem 6.(30 pts) A matrix A is given below.

$$A = \begin{bmatrix} 2 & 1 & 2 & 1 & 1 \\ 0 & 2 & -4 & 0 & 1 \\ 4 & 3 & 2 & 1 & -1 \\ 3 & 4 & -2 & 1 & 1 \end{bmatrix}$$

a) (15 pts) Find a basis for the row space and for the column space of A.
b) (10 pts) Find a basis for the null space of A.
c) (5 pts) Find the rank and the nullity of A.

Problem 7.(30 pts) Find the inverse of the given matrix or show that it does not exist.

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$