

MATH 0280 Final Examination, Sample 2

Problem 1. (15 pts)

Suppose A is a 4×4 matrix, and $\det A = 2$. Find $\det(A \cdot B^{-1})$, where

$$B = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 4 & 7 \end{bmatrix}$$

Problem 2. (15 pts)

A linear transformation F from \mathbb{R}^2 to \mathbb{R}^2 is defined as the projection onto the x -axis, followed by the rotation counterclockwise by $\frac{\pi}{3}$, followed by the projection onto the y -axis. Find the matrix of F .

Problem 3. (15 pts)

Determine if the vector v is in the span of the vectors u_1 , u_2 , and u_3 , where

$$v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 5 \\ 6 \\ 11 \end{bmatrix}$$

Problem 4. (15 pts)

a) Find an orthonormal basis of

$$W = \text{span}\left\{ \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix} \right\}$$

b) Find the projection onto W of the vector $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

Problem 5. (15 pts)

Matrix A is given:

$$A = \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}$$

- a) Find the characteristic polynomial of A .
- b) Find a matrix P such that $P^{-1}AP$ is diagonal, or show that it is not possible.

Problem 6. (25 pts)

Matrix A is given:

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 4 & 5 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

- a) Find a basis of $col(A)$.
- b) Find a basis of $row(A)$.
- c) Find a basis of $null(A)$.
- d) Find rank and nullity of A .