MATH 0280 FINAL EXAMINATION - Sample 1

Problem 1. a) Use **Gauss-Jordan elimination** (reduced row echelon form) to solve the system of linear equations

$$\begin{cases} \quad\ \ y \ -6z \ +5w = -7 \\ x \qquad\ \ +2z \ \ -w = 5 \\ 3x \ +y \qquad\ \ +2w = 8 \end{cases}$$

or explain why the system is inconsistent. If the system is consistent, write down the solution in a vector form. NO CREDIT will be given, if **any other method** is used.

b) For the matrix $A = \begin{pmatrix} 0 & 1 & -6 & 5 \\ 1 & 0 & 2 & -1 \\ 3 & 1 & 0 & 2 \end{pmatrix}$,

i) find a basis of the column space of A;

ii) find a basis of the row space of A;

iii) Determine rank and nullity of A;

iv) Determine if the vector $\mathbf{v} = \begin{bmatrix} -7 \\ 5 \\ 8 \end{bmatrix}$ is in $col(A)$.

Give detailed explanation.

Problem 2. a) Determine if the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 4 & 4 & 5 \\ 1 & 1 & 1 \end{pmatrix}$

is invertible. If A is invertible, find the inverse matrix A^{-1}. If A is not invertible, explain why.

b) Determine if the vectors

$$\mathbf{v_1} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \text{ and } \mathbf{v_3} = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix},$$

form a basis of $\mathbb{R}^3$ or not. Explain your conclusion in details.

Problem 3. Find the standard matrix of the linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$, that stretches a vector by a factor of 8 in the $x-$coordinate, then reflects it about the line $y = x$, and then rotates a vector clockwise by $30°$ about the origin.

Problem 4. The matrix A is given by

$$A = \begin{pmatrix} 0 & 3 & 6 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

a) Find all eigenvalues of A.

b) Find a basis for each eigenspace of A.

c) Find the algebraic and geometric multiplicities for each eigenvalue of A.

d) Determine if A is diagonalizable. If it is, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.
(You DO NOT have to find P^{-1})

Problem 5. If $W = span\,\{[1,0,-1,1,0],\ [3,1,0,0,1],\ [-1,2,3,1,-2]\}$,

a) Find a basis for the orthogonal complement $W^{\perp}$ of W.

b) Use Gram-Schmidt process to construct an orthogonal basis of W.