Problem 1. a) Use Gauss-Jordan elimination (reduced row echelon form) to solve the system of linear equations

\[
\begin{align*}
y - 6z + 5w &= -7 \\
x + 2z - w &= 5 \\
3x + y + 2w &= 8
\end{align*}
\]

or explain why the system is inconsistent. If the system is consistent, write down the solution in a vector form. NO CREDIT will be given, if any other method is used.

b) For the matrix

\[
A = \begin{pmatrix}
0 & 1 & -6 & 5 \\
1 & 0 & 2 & -1 \\
3 & 1 & 0 & 2
\end{pmatrix},
\]

i) find a basis of the column space of \(A\);

ii) find a basis of the row space of \(A\);

iii) Determine rank and nullity of \(A\);

iv) Determine if the vector \(v = \begin{pmatrix} -7 \\ 5 \\ 8 \end{pmatrix}\) is in \(\text{col}(A)\).

Give detailed explanation.

Problem 2. a) Determine if the matrix

\[
A = \begin{pmatrix}
2 & 0 & 1 \\
4 & 4 & 5 \\
1 & 1 & 1
\end{pmatrix}
\]

is invertible. If \(A\) is invertible, find the inverse matrix \(A^{-1}\). If \(A\) is not invertible, explain why.

b) Determine if the vectors

\[
v_1 = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}, \quad \text{and} \quad v_3 = \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix},
\]

form a basis of \(\mathbb{R}^3\) or not. Explain your conclusion in details.

Problem 3. Find the standard matrix of the linear transformation \(T\) from \(\mathbb{R}^2\) to \(\mathbb{R}^2\), that stretches a vector by a factor of 8 in the \(x\)-coordinate, then reflects it about the line \(y = x\), and then rotates a vector clockwise by \(30^\circ\) about the origin.
Problem 4. The matrix $A$ is given by

$$A = \begin{pmatrix} 0 & 3 & 6 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

a) Find all eigenvalues of $A$.

b) Find a basis for each eigenspace of $A$.

c) Find the algebraic and geometric multiplicities for each eigenvalue of $A$.

d) Determine if $A$ is diagonalizable. If it is, find an invertible matrix $P$ and a diagonal matrix $D$ such that $D = P^{-1}AP$.
(You DO NOT have to find $P^{-1}$)

Problem 5. If $W = \text{span} \{[1,0,-1,1,0], [3,1,0,0,1], [-1,2,3,1,-2]\}$,

a) Find a basis for the orthogonal complement $W^\perp$ of $W$.

b) Use Gram-Schmidt process to construct an orthogonal basis of $W$. 