Sample Final Exam 5

No phones, calculators, notes, or books.

Complete the following questions. Show all your work (no work = no credit!). Simplify your answers when possible. Your handwriting must be neat, clear, and visible (so use rather soft pencil than hard).

10 problems, 100 points.

1. (10 points) Let

$$P(1,0,2), \quad Q(3,1,0), \quad R(0,2,1).$$

   (a) Find a scalar equation of the plane that passes through the points P, Q, and R.

   (b) Find a parametric equation for the line through P and Q.

   (c) Find the distance from the origin to the plane found in part (a).

2. (10 points) Find the curvature of the curve

$$\vec{r}(t) = \langle t^2,\ 2t+4,\ t^3+4 \rangle$$

at the point $P = (1,2,3)$. Simplify your answer.

3. (10 points) Find and classify all critical points of the function

$$f(x,y) = \tfrac{1}{2}x^4 - xy + y^2.$$

4. (10 points) Find the absolute maximum and the absolute minimum values of the function

$$f(x,y,z) = x + y + 2z,$$

subject to the constraint

$$x^2 + y^2 + 3z^2 = 30.$$

5. (10 points) A lamina with the density $\rho(x,y) = 2x - y$ occupies a parallelogram with vertices $A(0,0)$, $B(3,1)$, $C(4,3)$, and $D(1,2)$. Use the change of variables $x = 3u + v$, $y = u + 2v$ to find the mass of the lamina.

6. (10 points) Evaluate $\displaystyle\iiint_E z\,dV$, where E is the hemispherical region that lies above the xy-plane and below the sphere $x^2 + y^2 + z^2 = 4$.

7. (10 points) Set up an iterated integral for the area of the part of the surface $z = x^2 y$ that lies above the triangle on the xy-plane with vertices $(0,0)$, $(2,0)$, and $(4,2)$. **DO NOT EVALUATE.**

8. (10 points)

   (a) (5 pts) Determine if the vector field

   $$\vec{F} = \langle (1+xy)e^{xy},\, x^2 e^{xy} \rangle$$

   is conservative.

   (b) (5 pts) Evaluate

   $$\int_C (1+xy)e^{xy}\, dx + x^2 e^{xy}\, dy,$$

   where C is the part of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$.

9. (10 points) Evaluate $\oint_C \vec{\mathsf{F}} \cdot d\vec{\mathsf{r}}$, where

   $$\vec{\mathsf{F}} = yz\,\vec{\mathsf{i}} + 2z\,\vec{\mathsf{j}} + 3y\,\vec{\mathsf{k}} = \langle yz,\, 2z,\, 3y \rangle,$$

   and C is the curve of intersection of the plane $2x + z = 4$ and the cylinder $x^2 + y^2 = 9$, oriented *clockwise* as viewed from above.

10. (10 points) Calculate the flux of the vector field

   $$\vec{\mathsf{F}} = \langle xe^y,\, xy - e^y,\, x(2-x) \rangle$$

   across the sphere $x^2 + y^2 + z^2 = 9$, oriented outward.