

1. (20 pts) Answer the following

(a) (3 pts) Determine the numerical value of $\tan\left(\arcsin\left(\frac{4}{7}\right)\right)$.

(b) (7 pts) A bacteria culture grows with a constant relative growth rate. After 2 hours there are 80 bacteria and after 5 hours the count is 5,120. Find an expression for the population after t hours.

(c) (5 pts) Determine the inverse of $f(x) = \frac{3x}{2x+3}$.

(d) (5 pts) If $f(x) = x^3 + 4x - 5$, then $f(1) = 0$. Determine $(f^{-1})'(0)$.

4. (5 pts each=40 pts) Differentiate the following. Do not simplify your answer.

(a) $f(x) = \sin^5(3x)$

(b) $y = \frac{3e^{2x}}{x^2 + 1}$

(c) $f(x) = \ln(4x + 3 \cos(2x))$

(d) $y = \arctan(2^x)$

$$(e) f(x) = \sqrt[3]{x - 3x^2 + 4x^3}$$

$$(f) y = (7x + 3)^4(x^2 + 1)^3$$

$$(g) f(x) = \frac{1000}{(1000 + 10e^{-0.1x})^2}$$

$$(h) y = \int_{\pi/4}^{x^2} \cos^4(6u) \sin^4(6u) du$$

5. (20 pts) Determine the limit (show all work).

$$(a) \lim_{x \rightarrow 1} \frac{\ln(1 + 4 \ln(x))}{x^2 - 1}$$

$$(b) \lim_{x \rightarrow 0} (1 + \arcsin(2x))^{1/x}$$

$$(c) \lim_{x \rightarrow 3^-} \frac{|x^2 - 5x + 6|}{x - 3}$$

$$(d) \lim_{x \rightarrow \infty} [\ln(4x^2 - 3x + 1) - \ln(x^2 + 7x + 80)]$$

6. (10 pts) 40π in³ of dough is being rolled so that it remains cylindrical. The length is increasing at $1/2$ inches per second. At what rate is the radius changing when the length is 10 inches?

7. (10 pts) Determine the range (minimum to maximum values) of the function $f(x) = x^2\sqrt{100 - x^2}$ on the interval $[5, 10]$.

8. (20pts) Given the function $f(x) = 8x^2e^{-x}$.

(a) (5 pts) State the exact points (x and y coordinates) where the function has any local maximum or local minimum values.

(b) (5 pts) State the values of x where $f(x)$ changes concavity

(c) (4 pts) $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$

(d) (6 pts) Graph the function $f(x) = 8x^2e^{-x}$ showing parts (a)-(c).

9. (15 pts) A farmer has a total of 750 ft of fence. He wants to enclose a rectangular area and then divide it into four parts by running fence parallel to one side of the rectangle. What is the largest possible total area of the four pens?

10. (5 pts each=40 pts) Integrate the following.

$$(a) \int_3^4 \frac{x}{\sqrt{25-x^2}} dx$$

$$(b) \int 3x^4 \ln(x) dx$$

$$(c) \int \cos^3(x) dx$$

$$(d) \int \frac{2x+3}{2x^2+6x+5} dx$$

$$(e) \int \arcsin(x) dx$$

$$(f) \int \sin^2(3x) dx$$

$$(g) \int \sec^3(x) \tan(x) dx$$

$$(h) \int \frac{3}{\sqrt{1-4x^2}} dx$$