**MATH 220-Analytic Geometry & Calculus I**

**Final Exam**

**Wednesday, April 23, 2014 – 10:00 -11:50am**

| **Last name:** **First name:** | People soft #: |
|---|---|

**Please Circle your instructor:** Borisov Brucato Burns Xiong Young

**Please circle your lecture time:** 9:00 10:00 11:00 12:00 1:00 2:00 3:00

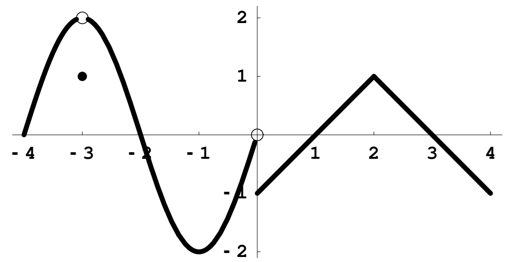
**This test consists of 13 questions. For full credit show all your work.**

| **Question** | **Points** | **Out of** |
|---|---|---|
| 1 | | 12 |
| 2 | | 16 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 12 |
| 7 | | 20 |
| 8 | | 10 |
| 9 | | 18 |
| 10 | | 12 |
| 11 | | 10 |
| 12 | | 12 |
| 13 a-c | | 24 |
| 13 d-f | | 24 |
| **Total** | | **200** |

1. (12 pts) For the function $f(x)$, graphed here,

**a**) (4 pts) Find the following limits. If the limit does not exist, please explain why not.

(i) $\lim_{x \to -3} f(x)$ (ii) $\lim_{x \to 0} f(x)$




**b**) (2 pts) At what value(s) of $x$ is $f(x)$ discontinuous?

**c**) (3 pts) At what value(s) of $x$ does $f(x)$ have a removable discontinuity? Please list the value of $y$ that you could use to redefine $f(x)$ so that $f(x)$ is continuous at the removable discontinuity.

**d**) (3 pts) At what value(s) of $x$ is $f(x)$ not differentiable?

2. (16 pts) Find the following limits, if the limit exists. If the limit does not exist, explain why.

(a) $\lim_{x \to 1} \frac{|2x^2 + x - 3|}{x - 1}$

(b) $\lim_{x \to \infty} \frac{\sin^2 x}{x^2}$

$\lim_{x \to 0} \frac{e^{-2x} - 1 + 2x}{1 - \cos(3x)}$

(d) $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$

3. (10 pts) Find the derivative of the function $f(x) = \sqrt{1-2x}$ using the limit definition of the derivative. NO CREDIT will be given if Limit Definition is not used.

4. (10 pts) Let $f(x) = 2x - 2 - \cos x$. Use Intermediate Value Theorem to show that $f(x) = 0$ has at least one root in the interval $[0, \pi]$.

5. (10 pts) Use implicit differentiation to find the equation of the tangent line to the curve

$$xy^2 + \ln(2x + 1) = y$$

at the point (0,0).

6. (12 pt) A flood lamp is installed on the ground 20 feet from a vertical wall. A 6-foot-tall man is walking towards the wall at the rate of 5 ft/s. How fast is the tip of his shadow moving down the wall when he is 5 feet from the wall?

7. (20 pts) Differentiate the following functions. You don't have to simplify your answers.

(a) (5 pts) Find $y''$ for $y = \sec^3(2x)$.

(b) (5 pts) $y = \dfrac{\cos^{-1}\sqrt{-x}}{4^x + 1} + \tan^{-1} x \sin^{-1} x$.

(c) (5 pts) $y = \dfrac{x^2 e^{-2x}}{\sqrt{(1+3x)(x-4)}}$.

(d) (5 pts) $y = \displaystyle\int_{x^2}^{0} \sin(u)(2-4u)^8 du$.

8. a) (5 pts) Find the linear approximation of the function $f(x) = x^{\frac{2}{3}}$ at $a = 8$ and use the linearization to approximate $f(7.9)$.

b) (5 pts) Use **Newton's Method** with the initial approximation $x_1 = 1$ to find the second approximation $x_2$ to the solution to the equation

$$e^{-2x} = \sqrt{x} + x$$

You don't have to simplify your result.

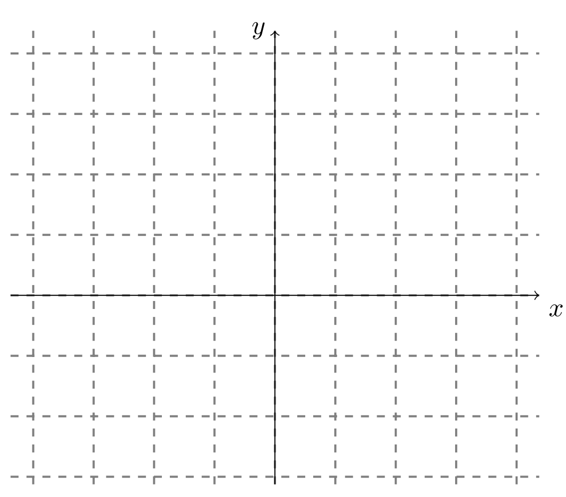
9. (18 pts) Given the function $f(x) = \dfrac{x^2}{x^2+3}$,

(a)(3 pts) Find its $x-$ and $y-$ intercepts, identify all vertical and horizontal asymptotes of the graph of this function if there exists.

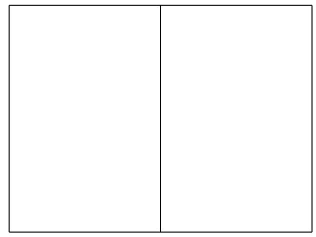
(b) (6 pts) Find the critical numbers of $f(x)$ and give the increasing and decreasing intervals. Determine whether the critical numbers are local minimizers, local maximizers or neither.

(c) (5 pts) Find the intervals where $f(x)$ are concave upward and concave downward .

(d) (4 pts) Sketch the graph of the function $f(x)$ representing clearly the information gathered above using the axis provided below.



10. (12 pts) A large rectangular area is to be fenced off as in the diagram below (a large rectangle divided into two smaller rectangles). The fence used to divide the space costs \$10 per foot and the fence used for the perimeter costs \$15 per foot. If the total budget for the project is \$6000, what are the dimensions which yield the largest area?

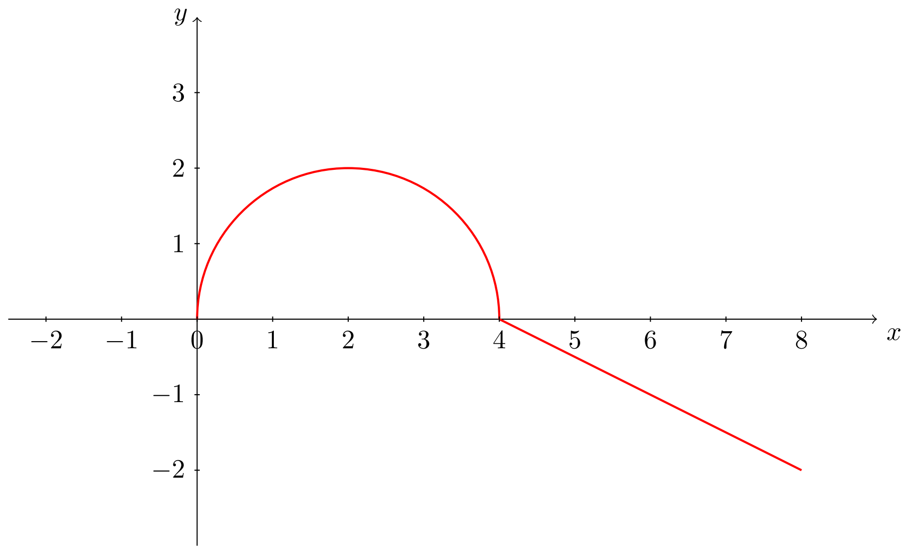


11. (10 pts) A particle moves along a straight line with the velocity $v(t) = t - \frac{16}{t^3}$ ft/s.

(a) ( 4 pts) Find the total displacement of the particle over the time interval $[1, 4]$.

(b) ( 6 pts) Find the total distance traveled by the particle over the interval $[1, 4]$.

12. (12 pts) Let $A(x) = \int_0^x f(t)dt$ where $f(t)$ is shown in the graph below. Note that the graph consists of a semicircle and a straight line segment.




(a) (4 pts) Find $A(4)$ and $A(8)$.

(b) (2 pts) Find $A'(2)$.

(c) (3 pts) Find all critical points of $A(x)$ in the interval $(0, 8)$ and classify each of them as a local maximum, local minimum, or neither.

(d) (3 pts) Determine the interval(s) on which $A(x)$ is concave up.

13. (48 pts, 8 pts each) Evaluate the following integrals.

(a) $\displaystyle\int \sqrt[3]{x^2} + \frac{1}{1+x^2} + e^2\,dx$

(b) $\displaystyle\int_{-4}^{4} x\sqrt[3]{x+4}\,dx$

(c) $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

(d) $\int_0^{\pi/4} 4\cos^2(2x)\,dx$

(e) $\int \sin^{2014} x \cdot \cos^3 x\,dx$

(f) $\int \frac{x}{e^{2x}}\,dx$