

MATH 0200 - Prep for Scientific Calculus
SAMPLE FINAL EXAM 3

Exam length: 1 hour 50 minutes

1. (10 points)

Solve the following equation.

$$|-2x + 1| - 5 = 10$$

2. (10 points)

Find the center, the foci, the length of the major axis, and the length of the minor axis for the ellipse defined by the equation

$$x^2 + 4x + 25y^2 - 50y = -4.$$

3. (10 points)

Find an equation for the line passing through the point $(3, 2)$ and parallel to the line $y = -x - 16$.

4. (10 points)

Find the vertex of the parabola $y = -3x^2 - 6x + 9$ and determine whether it is a minimum or maximum.

5. (10 points)

Find all numbers x such that the following equation holds.

$$\log_3(8x - 7) = 2$$

6. (40 points)

Solve these equations for all possible real values of x .

(a) $e^{2x} + 2 = 8 - e^x$

(b) $3^x = 27$

(c) $4 \log_3(5x - 3) = 12$

(d) $\log_3(4x) - \log_3(x - 5) = 2$

7. (10 points)

Construct an example of a rational function whose graph in the xy -plane has as vertical asymptotes the lines $x = 3$ and $x = 5$ and as an oblique asymptote the line $y = 2x - 3$.

8. (10 points)

If \$15,000 is deposited in a saving account with an annual interest rate of 7% compounded continuously, how long will it take for this amount to double?

9. (10 points)

On a twelve hour clock, find the angle in degrees between the hour hand and the minute hand at 2:27. Your answer should be between 0° and 180° .

10. (10 points)

Suppose

$$\frac{3\pi}{2} < \theta < 2\pi \quad \text{and} \quad \sin \theta = -0.6.$$

Evaluate the values of the other five trigonometric functions at θ .

11. (10 points)

Suppose a 40-foot ladder is leaning against a wall, making a 45° angle with the ground (measured from a perpendicular line from the base of the ladder to the wall).

(a) How high up the wall is the end of the ladder?

(b) How far from the wall is the base of the ladder?

12. (20 points)

Find the exact values for the indicated quantities if we are given that

$$\cos \frac{\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4}.$$

(a) $\sin \left(\frac{\pi}{12} \right)$

(b) $\sin \left(\frac{5\pi}{12} \right)$

(c) $\cos \left(\frac{5\pi}{12} \right)$

(d) $\tan \left(\frac{5\pi}{12} \right)$

(e) $\sin \left(\frac{\pi}{24} \right)$

(f) $\cos \left(\frac{\pi}{24} \right)$

13. (20 points)

Evaluate the following expressions.

(a) $\cos^{-1}(\cos(3\pi))$

(b) $\sin^{-1}\left(\sin\left(\frac{6\pi}{7}\right)\right)$

(c) $\cos^{-1}(\cos(40^\circ))$

(d) $\sin^{-1}(\sin(70^\circ))$

(e) $\sin\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$

(f) $\cos\left(\sin^{-1}\left(\frac{1}{4}\right)\right)$

(g) $\cos(\tan^{-1}(-4))$

(h) $\sin\left(-\sin^{-1}\left(\frac{3}{13}\right)\right)$

14. (10 points)

[6] Suppose t is such that $\cos^{-1} t = \frac{\pi}{17}$. Evaluate the following expressions:

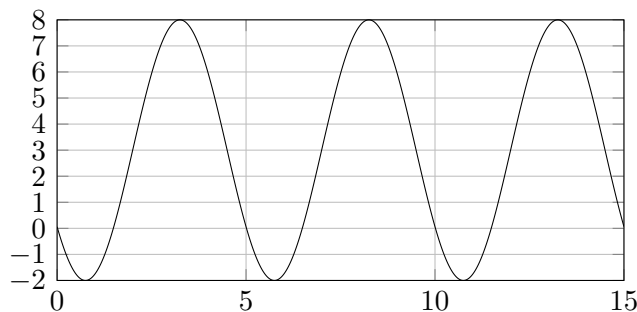
(a) $\cos^{-1}(-t)$

(b) $\sin^{-1}(t)$

(c) $\sin^{-1}(-t)$

15. (10 points)

Find a function that models the periodic behavior described by the graph below. You need to use the fact that the first minimum occurs at $x = \frac{3}{4}$.



16. (10 points)

Suppose the wind at airplane heights is 40 miles per hour (relative to the ground) moving 15° North of East. Relative to the wind, an airplane is flying at 450 miles per hour 20° South of the wind. Find the speed and direction of the airplane relative to the ground.

17. (10 points)

Find a number t such that $0 \leq t < \pi$ and the vectors $\vec{u} = \langle 3 \sin t, 4 \rangle$ and $\vec{v} = \langle 6, 3 \rangle$ are perpendicular.

18. (20 points)

Perform the indicated operation and write each of the following expressions in the form $a + bi$, where a and b are real numbers.

(a) $(5 + 7i)(4 + 6i) =$

(b) $\frac{4 + 3i}{5 - 2i} =$

(c) $(\sqrt{11} + \sqrt{3}i)^2 =$

(d) $\overline{(\sqrt{7} + \sqrt{3}i)} =$